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OF THE

ROYAL ASTRONOMICAL SOCIETY.

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No. 1

Dr. J. W. L. GLAISHER, M.A., F.R.S., PRESIDENT, in the Chair.

Professor W. W. Campbell, Director of the Lick Observatory,
San José, California, U.S.A. ;

Professor J. Scheiner, Astrophysical Observatory, Potsdam,
Germany ; and

M. Ch. Trépiéd, Director of the Observatory, Algiers,

were balloted for and duly elected Associates of the Society.

Spencer Lavington Fletcher, 38 Lammas Park Road, Ealing,
W. ; and

Professor Monroe B. Snyder, Director of the Observatory,
Philadelphia, Pa., U.S.A.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Ashton Charles Allen, 34 Kensington Palace Mansions, De Vere Gardens, W. (proposed by Rev. C. D. P. Davies) ;

John Hind Bell, *Nautical Almanac* Office, 3 Verulam Buildings, Gray's Inn, W.C. (proposed by A. M. W. Downing) ;

Walter Ernest Besley (H.M. Civil Service), 75 The Chase, Clapham Common, S.W. (proposed by F. W. Levander) ;

- James Henry Booth, F.R.G.S., Schoolmaster, 32 Sunny Road, Southport (proposed by William Banks) ;
 Arthur Thomas Cooper, A.M.Inst.E.E., Rivenook, De Montford Island, Reading (proposed by C. Hutchinson) ;
 Joseph Alfred Hardcastle, 38 Eaton Square, S.W. (proposed by H. H. Turner) ;
 Charles Sumner Howe, Ph.D., Professor of Astronomy, Case Observatory, Cleveland, Ohio, U.S.A. (proposed by Ambrose Swasey) ;
 William Gylby Hutchinson, Rock Lodge, Roker, Sunderland (proposed by C. Hutchinson) ;
 G. F. Johns, Perth Observatory, Western Australia (proposed by W. E. Cooke) ;
 James William Lowber, M.A., Sc.D., Ph.D., LL.D., F.R.G.S., F.R.Met. Soc., 113 East 18th Street, Austin, Texas, U.S.A. (proposed by Captain M. C. Hepworth) ;
 H. Krauss Nield, Thorold Grove, Sale, Cheshire (proposed by Samuel Chatwood) ;
 Captain Michael Joseph O'Sullivan, Board of Trade Surveyor, 66 Fern Grove, Hartington Road, Sefton Park, Liverpool (proposed by H. G. Quirk) ;
 Charles Tallent Spencer, A.M.Inst.C.E., La Quinta, Lyon Road, Harrow (proposed by A. G. Fowler) ; and
 Rev. Charles Lakeman Tweedale, M.A., The Vicarage, Weston, near Ottley (proposed by R. Wilding).
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One hundred and ninety-one presents were announced as having been received since the last meeting, including amongst others :—

Tycho Brahe, *De Nova Stella* (facsimile edition), presented by the R. Academy of Sciences, Copenhagen ; *Harvard Annals*, vol. 28, part 2 (spectra of bright southern stars), presented by the Observatory ; *Lick Observatory Publications*, vol. 5 (Hussey's measures of double stars), presented by the Observatory ; *Observations of the total eclipse of 1898 January 22*, presented by Sir N. Lockyer ; *Atlas photographique de la lune*, par MM. Loewy et Puiseux, presented by the Paris Observatory ; *Carte photographique du Ciel* (22 charts), presented by the Paris Observatory ; *Annals of the Astrophysical Observatory of the Smithsonian Institution*, vol. 1, presented by the Institution ; *Photographs of the great Comet of 1901*, presented by the Perth Observatory, Western Australia.

Note respecting the Printing of Tabular Matter in the Society's Publications.

The Secretaries wish to draw the attention of Fellows who submit papers for publication in the *Memoirs* or *Monthly Notices* to the importance of drawing up tabular matter so as to fit the size of page as nearly as possible. Care in this respect will avoid the necessity which has sometimes arisen of printing tables the wrong way of the page. For example, a convenient form in which observations of double stars may be printed in the *Memoirs* will be found in *Mem.* vol. liii. pp. 294-307, and similarly for the *Monthly Notices* in *M.N.* vol. lxi. pp. 486-500. The form in the *Monthly Notices* may be taken as showing the maximum number of columns which can be printed on a page. As a rule, notes are best printed at the bottom of the page or at the end of a paper.

[SECRETARIES.]

Determination of Dr. Küstner's Magnitude Equation from Comparison of his Meridian Observations in Zones $+24^{\circ}$ to $+27^{\circ}$ with Measures of Photographic Plates taken at the University Observatory, Oxford. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Dr. F. Küstner, Director of the Bonn Observatory, has embarked on a fine piece of work which he describes in the following terms: the formation of a catalogue of a number of stars of all magnitudes from the 4th to the 10th, uniformly distributed over the sky, reduced to the system of the Fundamental Catalogue of the *Astronomische Gesellschaft*, special attention being given to the determination and elimination of the magnitude equation in both R.A. and declination, and both of the fundamental catalogue and of the observer himself. Two parts of this work have already appeared.* The first gives a catalogue of 4,070 stars in zones 0° to $+18^{\circ}$, and the second a catalogue of 4,292 stars in zones $+18^{\circ}$ to $+36^{\circ}$. The zones under survey at the University Observatory, Oxford, as part of the astrographic chart work extend from $+24^{\circ}$ to $+32^{\circ}$ (plate centres $+25^{\circ}$ to $+31^{\circ}$), and the zones with centres $+25^{\circ}$ $+26^{\circ}$ $+27^{\circ}$ are complete, while those with centres $+28^{\circ}$ and $+29^{\circ}$ will probably be completed by the end of this year (thirty plates are yet unfinished). There is thus considerable material for

* *Veröffentlichungen der königl. Sternwarte zu Bonn*, Nos. 4 and 5.

comparison with Dr. Küstner's observations already, though a complete comparison cannot be made for a year or two. Since Dr. Küstner is directing special attention to the magnitude equation, which he investigates by the method of screens (cutting down the light of the bright stars for some transits and leaving it unobscured in others), it is of interest to give at once the totally independent determination afforded by photographic measures, which can be made with very little trouble.

2. In former papers (*Monthly Notices*, vol. lvii. p. 473, and vol. lx. p. 3) the magnitude equation of the Cambridge meridian observations was determined by comparison with Oxford photographic measures. The variation per magnitude was $0^{\circ}023$ for stars brighter than magnitude 8.0, but much more rapid (about $0^{\circ}100$) for fainter stars. In a note to the second paper (vol. lx. p. 11) it was stated that if the photographic method of determining magnitude equation were accepted as correct, the screen method seemed liable to underestimate the amount of the equation owing to difficulty in assigning the photometric value of the screen. It would have been interesting if the present comparison could have thrown further light on this point; but, unfortunately for this particular purpose, Dr. Küstner's magnitude equation is very small—practically zero. He obtains coefficients in R.A. of opposite signs in the two volumes published: for stars in zones 0° to $+18^{\circ}$ the result is $+0^{\circ}0070$; for zones $+18^{\circ}$ to $+36^{\circ}$ the result is $-0^{\circ}0041$; and it may be added that before beginning this work he got from the Victoria comparison stars the value $-0^{\circ}0040$. The result obtained from the subjoined comparison is $+0^{\circ}009$.

3. The comparison was effected as follows. In the course of the ordinary work at Oxford those stars which have been observed on the meridian at Cambridge, as part of the great catalogue of the *Astronomische Gesellschaft*, are used to get the plate constants. These being obtained, the residuals for the individual stars, in the sense Oxford—Cambridge, are entered in ledgers; and the mean of these residuals (for two or more separate plates as the case may be) is for the present purpose taken as the correction to the Cambridge place. The Cambridge places are given in the original catalogue for epoch 1875.0, but for use in our work they have been brought up to 1900.0, so that a comparison with Küstner (1900.0) is easily made. Adding then Oxford—Cambridge to Cambridge—Küstner we get Oxford—Küstner. The Cambridge places are thus nearly, but not quite, eliminated. Their influence remains in the means for all stars on a plate; if there were, for instance, a systematic error depending on R.A. running through the Cambridge Catalogue, this would remain in the Oxford photographic places. But since stars of all magnitudes would be equally affected this would not matter for our present purpose. The differences Oxford—Küstner for zones $+24^{\circ}$ to $+27^{\circ}$, formed in this way, were grouped according to magnitude (as given by Küstner) and means taken as below. It

should be particularly remarked that *no observations made with screens were included in the comparison*, which is thus altogether independent of screens.

Comparison of Oxford (Photographic) — Küstner (Visual)
Zones + 24° to + 27° only.

Limits of Mag.	Adopted Mean Mag.	No. of Stars.	Oxford — Küstner.		O — K Corrected.	
			R.A. s	Dec. "	R.A. s	Dec. "
7.1 to 8.0	7.5	49	+ 0.138	— 0.08	+ 0.014	+ 0.11
8.1 to 8.5	8.3	52	+ .116	— 0.24	— 0.015	— 0.13
8.6 & 8.7	8.65	35	+ .132	— 0.22	— 0.003	— 0.15
8.8 & 8.9	8.85	44	+ .132	— 0.05	— 0.004	0.00
9.0 & 9.1	9.05	68	+ .132	+ 0.02	— 0.006	+ 0.04
9.2	9.2	44	+ .136	0.00	— 0.004	+ 0.01
9.3	9.3	37	+ .148	+ 0.08	+ 0.007	+ 0.07
Over 9.3	9.6	38	+ .156	+ 0.04	+ 0.013	0.00
Means ...	8.8	46	+ .136	— 0.05		

4. It is clear that the magnitude equation is very small, especially in R.A. If we assume a formula

$$a + b \text{ (mag.} - 8.8)$$

to represent the differences, and find a and b by "least squares," regarding the means for the groups as of equal weight, we get numerical values for b as follows:—

$$\text{in R.A. } + 0^s.009 \quad \text{in Dec. } + 0''.11$$

Küstner's values are, from zones $+18^\circ$ to $+36^\circ$ (in which the zones $+24^\circ$ to $+27^\circ$, which form the basis of the above comparison, are included)

$$\text{in R.A. } - 0^s.0041 \quad \text{in Dec. } + 0''.064$$

while from zones 0° to $+18^\circ$ he got the results

$$\text{in R.A. } + 0^s.0070 \quad \text{in Dec. } + 0''.105$$

In the columns of the above table headed "O — K corrected" the residuals of the previous columns are given after subtracting $+ 0^s.136 + (m - 8.8) 0^s.009$ in R.A. and $- 0''.05 + (m - 8.8) 0''.11$ in Decl.

5. As indicating the general accuracy of both sets of results (Küstner and Oxford), the following table showing the number of differences of various values may be interesting.

Individual Value of Oxford—Küstner.

Value of Diff. s	In R.A.		No. of Cases.	Value of Diff. s	Value of Diff. "	In Decl.		No. of Cases.
	No. of Cases.	Value of Diff. s				No. of Cases.	Value of Diff. "	
-0'01	4	+0'16	25	-2'0	1	+0'1	22	
0'00	1	+0'17	23	-1'9	2	+0'2	19	
+0'01	2	+0'18	18	-1'3	1	+0'3	26	
+0'02	3	+0'19	15	-1'2	5	+0'4	17	
+0'03	3	+0'20	11	-1'0	2	+0'5	20	
+0'04	8	+0'21	8	-0'9	9	+0'6	9	
+0'05	6	+0'22	7	-0'8	8	+0'7	7	
+0'06	7	+0'23	5	-0'7	19	+0'8	5	
+0'07	15	+0'24	3	-0'6	15	+0'9	5	
+0'08	19	+0'25	6	-0'5	20	+1'0	5	
+0'09	16	+0'26	0	-0'4	29	+1'1	2	
+0'10	25	+0'27	1	-0'3	20	+1'2	4	
+0'11	17	+0'28	1	-0'2	35	+1'3	4	
+0'12	24	+0'29	0	-0'1	24	+1'4	1	
+0'13	34	+0'30	2	0'0	31			
+0'14	28	+0'31	1					
+0'15	28	+0'34	1					

General Conclusion.

Dr. Küstner's magnitude equation as determined by a comparison with Oxford photographic measures is in substantial agreement both in R.A. and Decl. with that found by him, using the method of screens ; but the quantities are too small to settle the question whether the screen method is apt to give values which should be multiplied by a factor, as suggested in a former paper.

On Periodic Orbits in the Neighbourhood of Centres of Libration.
By H. C. Plummer, M.A.

1. It is well known that there are five exact solutions of the problem of three bodies in each of which the bodies preserve an unvarying configuration which revolves with a uniform velocity. It is also known that when the third body is of infinitesimal mass compared with the other two, it can describe small periodic orbits in the vicinity of the points where exact solutions exist. These points have been given by Gylden the name of *centres of libration*, and a body describing an orbit of the kind indicated

has been called by Professor Darwin an *oscillating satellite*. Such orbits have been investigated recently by Dr. Charlier in a very interesting and suggestive paper.* The method which he has adopted is extremely simple and familiar, and several of his results had been given previously by Professor G. H. Darwin, in whose work the subject only presented itself incidentally, and by Mr. Moulton in his paper on the origin of the *Gegenschein*.† I wish here to discuss some of Dr. Charlier's results in a slightly more general manner. It is true that the direction in which greater generality can be secured without increased difficulty will lead us away from the problem of three bodies. But it is possible that the conclusions which follow will prove interesting in themselves, that they may conceivably find an application to some physical problem of a totally different order, and that something more may be learnt with regard to their general character and limitations. At any rate any attempt to draw wider attention to a subject which presents such a promising field for investigation may perhaps be justified.

2. Let us consider the motion of a particle in a rotating field of force. The forces are to be derived from a complete potential function Ω , the precise form of which need not at present be defined except as some uniform function of x and y , the coordinates referred to rotating axes. Then the equations of motion of the particle may by a suitable choice of units be expressed in the form

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}$$

As is well known these apply in particular to the motion of a body of infinitesimal mass under the influence of two bodies revolving in circles about their centre of gravity. Now let a centre of libration be defined as a point whose coordinates (x, y) satisfy the relations

$$\frac{\partial \Omega}{\partial x} = \frac{\partial \Omega}{\partial y} = 0$$

At such a point equilibrium, which may be either stable or unstable, is possible. Now let a surface be defined by the equation $z = \Omega$, and let the ordinary notation be adopted, viz.

$$\frac{\partial z}{\partial x} = p \quad \frac{\partial z}{\partial y} = q$$

$$\frac{\partial^2 z}{\partial x^2} = r \quad \frac{\partial^2 z}{\partial x \partial y} = s \quad \frac{\partial^2 z}{\partial y^2} = t$$

* *Meddelanden från Lunds Astronomiska Observatorium*, No. 18.

† *Astr. Journ.*, No. 483.

Now if in the equations of motion we substitute $x + \xi$, $y + \eta$ for x and y and consider (x, y) a fixed point, we obtain

$$\begin{aligned}\ddot{\xi} - 2\dot{\eta} &= \frac{\partial \Omega}{\partial x} + \xi \frac{\partial^2 \Omega}{\partial x^2} + \eta \frac{\partial^2 \Omega}{\partial x \partial y} + \dots \\ \ddot{\eta} + 2\dot{\xi} &= \frac{\partial \Omega}{\partial y} + \xi \frac{\partial^2 \Omega}{\partial x \partial y} + \eta \frac{\partial^2 \Omega}{\partial y^2} + \dots\end{aligned}$$

Hence if (x, y) is a centre of libration so that $p=q=0$ and if we neglect second and higher powers of ξ and η we may write

$$\begin{aligned}\ddot{\xi} - 2\dot{\eta} &= r\xi + s\eta \\ \ddot{\eta} + 2\dot{\xi} &= s\xi + t\eta\end{aligned}$$

Then $\xi = Ae^{\lambda t}$, $\eta = Be^{\lambda t}$ satisfy the equations provided

$$\begin{aligned}A(\lambda^2 - r) - B(2\lambda + s) &= 0 \\ A(2\lambda - s) + B(\lambda^2 - t) &= 0 \\ \text{i.e.} \quad (\lambda^2 - r)(\lambda^2 - t) + 4\lambda^2 - s^2 &= 0 \\ \text{or} \quad \lambda^4 + \lambda^2(4 - r - t) + rt - s^2 &= 0\end{aligned}$$

Stability depends on the nature of the roots of this equation.

3. Now let ρ_1 , ρ_2 be the principal radii of curvature at the corresponding point of the surface $z = \Omega$. Since $p = q = 0$ they are the roots of the quadratic

$$(rt - s^2)\rho^2 + (r + t)\rho + 1 = 0$$

Hence we may rewrite the preceding equation in λ^2 in the form

$$\lambda^4 + \lambda^2 \left(4 + \frac{1}{\rho_1} + \frac{1}{\rho_2} \right) + \frac{1}{\rho_1 \rho_2} = 0$$

By means of this equation the conditions of stable equilibrium at a centre of libration are immediately connected with the properties of the surface $z = \Omega$. For perfect stability it is necessary that both values of λ^2 shall be real and negative. Hence we must in the first place have both ρ_1 and ρ_2 of the same sign, the surface synclastic at the point, and z a real maximum or a real minimum. The second condition is that

$$4 + \frac{1}{\rho_1} + \frac{1}{\rho_2} > \frac{2}{\sqrt{\rho_1 \rho_2}}$$

If z is a maximum, ρ_1 and ρ_2 are positive, and the condition is necessarily fulfilled. If z is a minimum, ρ_1 and ρ_2 are negative, and the condition may be written

$$\frac{1}{\sqrt{-\rho_1}} + \frac{1}{\sqrt{-\rho_2}} < 2$$

Thus, while in a stationary field of force equilibrium is stable where the force function is a maximum only, in a rotating field the function Ω , which it must be remembered includes the rotation potential, may be either a maximum or a minimum, provided in the latter case a certain inequality is satisfied. On the other hand we may have one value of λ^2 negative and the other positive. This corresponds to curvatures of opposite sign or an anticlastic point on the Ω surface. In this case a mode of periodic motion is combined with an unstable mode which must sooner or later prove fatal to the stable form. Neither purely positive nor imaginary values of λ^2 can be admitted in a search for periodic orbits.

4. I consider first the far more interesting case in which two periodic solutions are possible. Let us assume

$$\xi = h \cos (mt - \alpha) \qquad \eta = k \cos (mt - \beta)$$

as a periodic solution of the equations

$$\begin{aligned} \ddot{\xi} - 2\dot{\eta} &= r\xi - s\eta \\ \ddot{\eta} + 2\dot{\xi} &= s\xi + t\eta \end{aligned}$$

The conditions obtained by substitution are

$$\begin{aligned} (r + m^2) h \cos \alpha + sk \cos \beta &= -2mk \sin \beta \\ (r + m^2) h \sin \alpha + sk \sin \beta &= 2mk \cos \beta \\ (t + m^2) k \cos \beta + sh \cos \alpha &= 2mh \sin \alpha \\ (t + m^2) k \sin \beta + sh \sin \alpha &= -2mh \cos \alpha \end{aligned}$$

Hence we obtain directly

$$\begin{aligned} (r + m^2) h \sin (\alpha - \beta) &= 2mk \\ sk \sin (\alpha - \beta) &= -2mk \cos (\alpha - \beta) \\ (t + m^2) k \sin (\alpha - \beta) &= 2mh \\ sh \sin (\alpha - \beta) &= -2mh \cos (\alpha - \beta) \end{aligned}$$

which are equivalent to

$$\begin{aligned} h^2(r + m^2) &= k^2(t + m^2) \\ s \tan (\alpha - \beta) &= -2m \\ 4m^2 \operatorname{cosec}^2 (\alpha - \beta) &= (r + m^2)(t + m^2) = 4m^2 + s^2 \end{aligned}$$

Hence the periodic orbit is obtained by combining two simple harmonic motions at right angles. These motions are characterised by a constant ratio of the amplitudes and a constant difference of phase for each centre of libration and assigned period. The orbits may be illustrated by Lissajou's figures for tuning-forks in unison. This is a very simple way of stating the

connection between the integration constants. The true arbitrariness are, of course, the epoch and the scale of the orbit. Except for these the character of the orbits is absolutely determined by the nature of the field of force.

5. The equation of the orbit can be written down at once in the form

$$\frac{\xi^2}{h^2} + \frac{\eta^2}{k^2} - \frac{2\xi\eta}{hk} \cos(\alpha - \beta) = \sin^2(\alpha - \beta)$$

It will be found that an important distinction must be drawn in what follows, according as equilibrium corresponds to a maximum or minimum value of Ω . The latter case may be considered first. Then both r and t are positive, and we may put

$$c^2 = h^2(r + m^2) = k^2(t + m^2)$$

and since

$$s(r + m^2)hk \sin^2(\alpha - \beta) = -4m^2k^2 \cos(\alpha - \beta)$$

we have also

$$-hk \cdot s \sec(\alpha - \beta) = c^2$$

The orbit becomes

$$(r + m^2)\xi^2 + (t + m^2)\eta^2 + 2s\xi\eta = 4m^2c^2 / (4m^2 + s^2)$$

and the axes and foci are given by

$$\frac{\xi^2 - \eta^2}{r - t} = \frac{\xi\eta}{s} = -\frac{c^2}{4m^2 + s^2}$$

The directions of the axes depend, then, on the form of the potential alone, and the major axis lies in the first and third quadrants if s is negative, in the second and fourth if s is positive. But for the surface $z = \Omega$

$$z + \delta z = \Omega + p\xi + q\eta + \frac{1}{2}(r\xi^2 + 2s\xi\eta + t\eta^2) + \dots$$

or

$$2\delta z = r\xi^2 + 2s\xi\eta + t\eta^2$$

is the indicatrix corresponding to a centre of libration. The axes and foci are given by

$$\frac{\xi^2 - \eta^2}{r - t} = \frac{\xi\eta}{s} = \frac{(2\delta z)^2}{s^2 - rt}$$

and $s^2 - rt = -\rho_1^{-1}\rho_2^{-1}$ is negative. Hence the major axes of the orbit and of the indicatrix are parallel. An alternative form of statement is that the axes of the orbit are parallel to the tangents to the lines of curvature at the corresponding point of the Ω surface.

6. The semi-axes a , b of the orbit are given by

$$\frac{1}{a^2} + \frac{1}{b^2} = (r+t+2m^2) (4m^2+s^2) / 4m^2c^2$$

$$\frac{1}{a^2 b^2} = (4m^2+s^2)^2 / 4m^2c^4$$

Hence a^2 and b^2 are the roots of the quadratic

$$(4m^2+s^2)^2 x^2 - (r+t+2m^2) (4m^2+s^2)c^2x + 4m^2c^4 = 0$$

$$\therefore 2(4m^2+s^2) \cdot x/c^2 = r+t+2m^2$$

$$\pm \sqrt{\{4m^4 + 4m^2(r+t-4) + (r+t)^2\}}$$

$$= r+t+2m^2 \pm \sqrt{\{4(s^2-rt) + (r+t)^2\}}$$

$$= 2m^2 - \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) \pm \left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

We may therefore write

$$a^2 = c^2 \left(m^2 - \frac{1}{\rho_1}\right) / (4m^2+s^2)$$

$$b^2 = c^2 \left(m^2 - \frac{1}{\rho_2}\right) / (4m^2+s^2)$$

where ρ_1 is the numerically smaller of the two radii of curvature, for these are both negative. Accordingly the eccentricity e is given by

$$e^2 = (\rho_2^{-1} - \rho_1^{-1}) / (m^2 - \rho_1^{-1})$$

and we notice that a circular orbit corresponds to an umbilic on the Ω surface.

7. We can now examine the initial conditions and the relation of our constant c to the constant C of the Jacobian integral

$$V^2 = 2\Omega - C$$

Let (ξ_0, η_0) be the position at the time $t=0$. Then since

$$\xi = h \cos(mt-a) \quad \eta = k \cos(mt-\beta)$$

we have

$$\xi_0 = h \cos a \quad \eta_0 = k \cos \beta$$

$$\dot{\xi}_0 = m h \sin a \quad \dot{\eta}_0 = m k \sin \beta$$

$$\therefore V^2 = \dot{\xi}_0^2 + \dot{\eta}_0^2 = m^2(h^2 + k^2) - m^2(\xi_0^2 + \eta_0^2)$$

But

$$2\Omega = 2\Omega_0 + r\xi_0^2 + t\eta_0^2 + 2s\xi_0\eta_0$$

where Ω_0 is the value of Ω corresponding to the centre of libration. If $2\Omega_0 = C_0$

$$C = C_0 - m^2(h^2 + k^2) + (r+m^2)\xi_0^2 + (t+m^2)\eta_0^2 + 2s\xi_0\eta_0$$

Thus for a given value of $C - C_0$, ξ_0 and η_0 must satisfy the equation of a certain ellipse which can be no other than the orbit already found. Hence by comparison we find

$$\begin{aligned} C - C_0 &= 4m^2c^2 / (4m^2 + s^2) - m^2(h^2 + k^2) \\ &= \frac{4m^2c^2}{4m^2 + s^2} - \frac{m^2c^2(2m^2 + r + t)}{(m^2 + r)(m^2 + t)} \\ &= m^2c^2(4 - r - t - 2m^2) / (4m^2 + s^2) \end{aligned}$$

Now if m_1^2 and m_2^2 are the roots of the quadratic giving m^2

$$m_1^2 + m_2^2 = 4 - r - t$$

or

$$(2m_1^2 + r + t - 4) + (2m_2^2 + r + t - 4) = 0$$

Hence $C - C_0$ clearly has opposite signs according to the period selected, that is, one period and one eccentricity correspond to an increase, and the other period and eccentricity to a decrease of the energy constant as compared with the value required for relative rest. Thus when stable equilibrium exists at a point where Ω is a minimum a given value of C determines the orbit uniquely. Dr. Charlier has noticed this in a particular case, but his proof is not of a general character. Another peculiarity is that one family of orbits is bounded by a set of "limiting curves," while the other is not.

8. The period of the orbit is $2\pi/m$. But the units have been so chosen that the period of rotation of the field of force is 2π . Therefore m may be defined as the ratio of the period of the rotating field to the synodic period of the orbit. The direction of motion is determined by the sign of

$$\begin{aligned} \xi\dot{\eta} - \dot{\xi}\eta &= -mhk \sin(\alpha - \beta) \\ &= -2m^2c^2 / (4m^2 + s^2) = -mab \end{aligned}$$

The angular momentum being negative, the orbits of both families are retrograde. If it is desired to write down the equation of an orbit in a form involving only the energy constant and quantities depending only on Ω , we are led to

$$\begin{aligned} (r + m^2)\xi^2 + (t + m^2)\eta^2 + 2s\xi\eta &= 4(C - C_0) / (4 - r - t - 2m^2) \\ &= 4n(C - C_0) \end{aligned}$$

where

$$\begin{aligned} n^{-2} &= (4 - r - t)^2 - 4(r^2 - s^2) \\ &= \left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)^2 + 8\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) + 16 \end{aligned}$$

It may be observed that it is the family of shorter period and smaller eccentricity which requires $C < C_0$.

9. Throughout §§ 5-8 it has been assumed that Ω is an absolute minimum at the centre of libration, and that accordingly

r and t are positive. This indeed happens to be the more interesting case in so far as the problem of three bodies is concerned because there the function Ω does not as a matter of fact possess a truly maximum value. But if Ω is an absolute maximum both r and t are negative. It will be convenient to denote the quadratic giving m^2 by $f(m^2)=0$. By substitution we obtain :

$$\begin{aligned} f(+\infty) & \text{ is positive.} \\ \left. \begin{aligned} f(-r) &= 4r - s^2 \\ f(-t) &= 4t - s^2 \end{aligned} \right\} & \text{ are negative.} \\ f(0) &= rt - s^2 \text{ is positive.} \end{aligned}$$

Hence $-r$ and $-t$ separate the two positive roots m_1^2 and m_2^2 . It is clear then that when Ω is a maximum $m_1^2 + r$ and $m_1^2 + t$ are negative, while $m_2^2 + r$ and $m_2^2 + t$ are positive if $m_2^2 > m_1^2$. Further, since

$$4 - r - t - 2m_1^2 > 0$$

we must have

$$4 - r - t - 2m_2^2 < 0$$

The orbits of shorter period, $2\pi/m_2$, are then similar to the orbits investigated in the case when Ω is a minimum, and they require that $C < C_0$. It is unnecessary to consider this family in further detail. It is only in the expressions for the semi-axes and eccentricity given in § 6 that it is necessary to interchange the two radii of curvature on account of the change of sign.

10. The family of orbits characterised by the longer period $2\pi/m_1$ possesses features, however, which require separate notice. In the place of $+c^2$ in the foregoing it is necessary to write $-c^2$. The formula given for the foci in § 5 shows that the major axis of a member of this family is at right angles to the corresponding axis of a short-period orbit. Further from § 8 it appears that the direction of motion is reversed and orbits of this family are direct. Since both c^2 and $(4 - r - t - 2m^2)$ have opposite signs in the two families we again have $C < C_0$. Consequently to any given value of $C < C_0$ one orbit of each family corresponds. Indeed the Jacobian integral suffices to prove that $C > C_0$; for otherwise since $2\Omega_0 > 2\Omega$, the velocity of the displaced body would be imaginary. Hence the extension of Dr. Charlier's statement given in § 7 of this paper, must be restricted to cases where Ω is a minimum. The expressions given for the semi-axes in § 6 require no alteration, although the radii of curvature are positive: for $(m_1^2 - \rho_1^{-1})$ and $(m_1^2 - \rho_2^{-1})$ must be negative, and therefore $\rho_1^{-1} > \rho_2^{-1}$. The most interesting features of the family of orbits considered here are undoubtedly the change from retrograde to direct motion and the rotation of the major axis through a right angle.

11. When Ω is a maximum at the centre of libration, we have seen that two orbits correspond to one value of C . Again,

through a point two orbits can be drawn, and for these the values of C will in general be different. There are, however, points where two orbits of different families, and the same C meet. The orbits will be

$$\begin{aligned}(m_1^2 + r)\xi^2 + (m_1^2 + t)\eta^2 + 2s\xi\eta &= +4n(C - C_0) \\ (m_2^2 + r)\xi^2 + (m_2^2 + t)\eta^2 + 2s\xi\eta &= -4n(C - C_0)\end{aligned}$$

Hence the locus of their intersection is

$$(m_1^2 + m_2^2 + 2r)\xi^2 + (m_1^2 + m_2^2 + 2t)\eta^2 + 4s\xi\eta = 0$$

Or since

$$\begin{aligned}m_1^2 + m_2^2 &= 4 - r - t \\ (4 + r - t)\xi^2 + (4 - r + t)\eta^2 + 4s\xi\eta &= 0\end{aligned}$$

This represents a pair of straight lines equally inclined to the axes of the orbit, and this is clear from the geometrical interpretation. For since C is the same for both orbits, the velocity in each at the point of intersection is the same. Hence the central perpendiculars on the tangents are equal, and it becomes evident that the above equation represents the locus of points where orbits of the two families intersect each other, and are equally inclined to the common radius vector. The locus will not be real unless

$$4s^2 + (r - t)^2 > 16$$

i.e.

$$\rho_1^{-1} \sim \rho_2^{-1} > 4$$

And unless this condition is satisfied corresponding members of the two families will not intersect, so that one orbit will lie wholly within the other.

12. When at a centre of libration the function Ω is neither an absolute maximum nor an absolute minimum, oscillating satellites may still exist. In this case only one class of orbit is possible, and even this cannot be permanently stable. The equation $f(m^2) = 0$ has one positive root and one negative. Since the equation

$$(2m_1^2 + r + t - 4) + (2m_2^2 + r + t - 4) = 0$$

still holds, the first expression on the left hand must be positive if $m_1^2 \sim 0 > m_2^2$. Hence $(2m_1^2 + r + t)$ is positive, and since $(m_1^2 + r)$ and $(m_1^2 + t)$ must be of the same sign, if an elliptic orbit exists they must both be positive. Hence $C < C_0$, and the orbits possess the same general features as those of shorter period when Ω is a minimum. An unstable form of motion arises immediately if $C > C_0$, and this would be the case if motion in the periodic orbit be unduly retarded. We should infer that excess rather than defect of relative velocity would be favourable to the duration of quasi-stable motion, and this may have some bearing on the theory of the detention of meteorites in the neighbourhood of a centre of libration. We have seen that in general the direction

of the major axis of an orbit depends on the sign of s . An exception, of course, arises when $s=0$. This point happens to occur in the application of this paragraph, and may properly be considered in this connection. The formula given in § 5 for the foci of the orbit shows that the axes are parallel to the rotating axes of coordinates, and that the major axis is parallel or perpendicular to the line joining the controlling bodies, according as $r-t < \text{or} > 0$. The only exception to this rule is in the case of long-period orbits corresponding to Ω a maximum, when the preceding inequalities are reversed.

13. About the special problem which suggested the foregoing slightly more general investigation, little need be said here, because it has been treated with some fullness by Dr. Charlier. But as I have had occasion to allude to the properties of the surface defined by the equation $z=\Omega$, it may not be out of place to consider briefly the nature of the surface determined by two bodies revolving in circles about their centre of gravity. The sum of the masses μ and ν is taken as the unit of mass, while the unit of time is such that the gravitational constant is unity and the unit of length the distance between the masses. Then the period $T=2\pi$, the angular velocity is unity, and we have

$$2\Omega = \mu \left(r_1^2 + \frac{2}{r_1} \right) + \nu \left(r_2^2 + \frac{2}{r_2} \right)$$

where r_1, r_2 are the distances from μ and ν respectively. Now Professor Darwin in his memoir * has drawn a typical series of the curves $2\Omega = \text{const.}$, and if these be regarded not merely as a family of curves with an arbitrary parameter, but as a set of contour lines of a surface, we easily get a fair notion of the general character of that surface. Much indeed can be gathered from the simplest reasoning. At the two points where $r_1=r_2=1$, it is easy to prove that 2Ω is a true minimum and equal to 3. The familiar topographical illustration will be useful here. At each of these minimum points there is a valley. The function Ω becomes infinite at the points where the masses are placed, but for the present purpose these points may be represented by the summits of mountains. The contour lines, which are evidently symmetrical with respect to the line joining the masses, must rapidly approach a circular form as they recede from the centres of force and indicate rising ground in all directions outwards. The additional features of the surface must clearly be three passes, one running between the mountains and connecting the valleys, and two others beyond the two mountains and separating them from the ground rising in the distance. Each of the three passes has its highest point on the line of symmetry, and these three points correspond to the three collinear centres of libration. Here the Ω surface is anticlastic, and, as indeed is evident on dynamical grounds, stable equilibrium is out of the question.

* *Acta Math.*, t. xxi. p. 113.

14. Although there cannot be any doubt as to the nature of these points, yet it may be permissible to consider them from another point of view. In what follows the origin is taken at the centre of gravity of the revolving bodies, the latter always lying on the rotating axis of x . Since

$$2\Omega = \mu \left(r_1^2 + \frac{2}{r_1} \right) + \nu \left(r_2^2 + \frac{2}{r_2} \right)$$

we have

$$\frac{\partial \Omega}{\partial r_1} = \mu \left(r_1 - \frac{1}{r_1^2} \right); \quad \frac{\partial \Omega}{\partial r_2} = \nu \left(r_2 - \frac{1}{r_2^2} \right)$$

Also for the collinear centres of libration

$$y = 0; \quad \frac{\partial \Omega}{\partial x} = 0$$

or

$$\frac{\partial \Omega}{\partial r_1} \cdot \frac{\partial r_1}{\partial x} + \frac{\partial \Omega}{\partial r_2} \cdot \frac{\partial r_2}{\partial x} = 0$$

Moreover

$$\begin{aligned} \frac{\partial \Omega}{\partial y} &= \frac{\partial \Omega}{\partial r_1} \cdot \frac{\partial r_1}{\partial y} + \frac{\partial \Omega}{\partial r_2} \cdot \frac{\partial r_2}{\partial y} \\ &= \frac{y}{r_1} \cdot \frac{\partial \Omega}{\partial r_1} + \frac{y}{r_2} \cdot \frac{\partial \Omega}{\partial r_2} \end{aligned}$$

Hence when $y = 0$

$$\begin{aligned} t &= \frac{\partial^2 \Omega}{\partial y^2} = \frac{1}{r_1} \cdot \frac{\partial \Omega}{\partial r_1} + \frac{1}{r_2} \cdot \frac{\partial \Omega}{\partial r_2} \\ s &= \frac{\partial^2 \Omega}{\partial x \partial y} = 0 \end{aligned}$$

We have now to treat separately the cases which arise.

(1) Between the masses μ and ν we have

$$r_1 + r_2 = 1; \quad \frac{\partial r_1}{\partial x} + \frac{\partial r_2}{\partial x} = 0$$

$$\therefore \frac{\partial \Omega}{\partial r_1} = \frac{\partial \Omega}{\partial r_2}$$

$$\therefore \mu \left(\frac{1}{r_1^2} - r_1 \right) = \nu \left(\frac{1}{r_2^2} - r_2 \right)$$

Hence clearly $r_1 > r_2$ if $\mu > \nu$. In this case

$$t = \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \cdot \mu \left(r_1 - \frac{1}{r_1^2} \right)$$

and is negative, for $r_1 < 1$.

(2) Beyond the mass μ we have

$$r_2 = r_1 + 1; \quad \frac{\partial r_1}{\partial x} = \frac{\partial r_2}{\partial x}$$

$$\therefore \frac{\partial \Omega}{\partial r_1} + \frac{\partial \Omega}{\partial r_2} = 0$$

$$\therefore \mu \left(\frac{1}{r_1^2} - r_1 \right) = \nu \left(r_2 - \frac{1}{r_2^2} \right)$$

Hence clearly $r_1 < 1$. In this case

$$t = \left(\frac{1}{r_1^2} - \frac{1}{r_2} \right) \cdot \mu \left(r_1 - \frac{1}{r_1^2} \right)$$

and is negative because $r_1 < 1 < r_2$, and therefore the first factor is positive and the last negative. The proof that t is negative in the case of the centre of libration lying beyond the mass ν can be inferred from the latter, as there is no restriction on the ratio of the masses μ and ν . Now we have

$$r = \frac{\partial^2 \Omega}{\partial x^2} = \mu \left(1 + \frac{2}{r_1^3} \right) + \nu \left(1 + \frac{2}{r_2^3} \right)$$

which is necessarily positive. Hence always for the three points considered

$$rt - s^2 < 0$$

and therefore the two roots of the quadratic in m^2 are of opposite signs. I have thought it worth while to give a definite proof of this fact because on this point Dr. Charlier says: "That is what I have found through numerical calculation for different values of μ , an algebraic demonstration of this fact seeming to be somewhat complicated."

University Observatory, Oxford:
1901 October 1.

A New Method of Interpolation. By T. C. Hudson, B.A.

1. Bessel's interpolation formula is

$$u_n - u_o = n \Delta'_{\frac{1}{2}} + \frac{n(n-1)}{1 \cdot 2} \frac{\Delta_o'' + \Delta_1''}{2} + \frac{n(n-1)(n-\frac{1}{2})}{1 \cdot 2 \cdot 3} \Delta_{\frac{1}{2}}''' \\ + \frac{(n+1)n(n-1)(n-2)}{1 \cdot 2 \cdot 3 \cdot 4} \frac{\Delta_o^{iv} + \Delta_1^{iv}}{2} + \dots \quad (1)$$

2. As it stands this formula is partially symmetrical with respect to the middle of the interval; but there is no symmetry

in the first three terms, namely, u_n, u_o and $n \Delta'_{\frac{1}{2}}$, and the symmetry of the last three does not present itself obviously to the eye. If, however, we replace

$$\frac{u_o + u_r}{2}, \frac{1}{1 \cdot 2}, \frac{\Delta_o'' + \Delta_r''}{2}, \frac{1}{1 \cdot 2 \cdot 3 \cdot 4}, \frac{\Delta_o^{iv} + \Delta_r^{iv}}{2}, \Delta_{\frac{1}{2}}', \Delta_{\frac{1}{2}}''', \frac{1}{2} - n, u_n$$

by $M, M'', M^{iv}, \Delta', \Delta''', m, U_{+m}$

equation (1) becomes

$$U_{+m} = M - \left(\frac{1}{4} - m^2\right)M'' + \left(\frac{1}{4} - m^2\right)\left(\frac{9}{4} - m^2\right)M^{iv} - \dots$$

$$- m\Delta' + m\left(\frac{1}{4} - m^2\right)\frac{\Delta'''}{6} - \dots$$

(a modification of Bessel's formula due, it appears, to Newton) which may be written

$$U_{+m} = [E] + [O]$$

where $[E] = M - \left(\frac{1}{4} - m^2\right)M'' + \left(\frac{1}{4} - m^2\right)\left(\frac{9}{4} - m^2\right)M^{iv} - \dots$

containing only *even* differences and *even* powers of m ,

and $[O] = -m\Delta' + m\left(\frac{1}{4} - m^2\right)\frac{\Delta'''}{6} - \dots$

containing only *odd* differences and *odd* powers of m .

In the E equation it will be seen that M takes its place naturally with M'', M^{iv} , &c., its coefficient obeying the same law as theirs, and that the original quantities may be justly regarded as belonging to the family of even differences.

3. Changing the sign of m , and observing that U_{-m} is merely u_{1-m} , we have

$$U_{-m} = [E] - [O].$$

By these substitutions we have now arrived at a pair of formulæ

$$\left\{ \begin{array}{l} U_{+m} = [E] + [O] \\ U_{-m} = [E] - [O] \end{array} \right\} \dots \dots \dots (2)$$

which possesses the practical, economic advantage of being symmetrical with respect to the middle of the interval.

4. The advantage consists in this, that when once $[E]$ and $[O]$ have been computed for the first half of the interval, and their sum has given the *first* half of the new quantities (u_n or U_{+m}), then their difference gives the *second* half without further computation. This means a saving of half the labour involved in a great part of the work of systematic interpolation. Moreover, the more new quantities we have to interpolate between the old data, the greater (relatively) becomes this economy. In particular, to take the opposite extreme, in the simplest case, that of "bisection," where we should want only midway quantities, we

observe that m , $[O]$, and the difference between the formulæ (2) would all vanish together—likewise the economy.

But in the actual case of the work of deriving the Moon's hourly places for the *Nautical Almanac* from twelve-hourly data throughout a whole year, the degree of subdivision of the original interval into twelve new ones is so great that the aforesaid economy becomes an important consideration.

5. For this reason I found it worth while, a few years ago, to make new tables for the interpolation of the Moon's hourly R.A. and Dec., in which case (known variously as "interpolation to twelfths" and as "interval twelve") we have the following values:—

$$\text{of } 12n \quad \text{viz. } \left\{ \begin{array}{cccccc} \text{I} & 2 & 3 & 4 & 5 \rightarrow & \\ \text{II} & \text{IO} & 9 & 8 & 7 \leftarrow & 6 \end{array} \right.$$

$$\text{of } 12m \quad \text{,, } \{ \pm 5 \pm 4 \pm 3 \pm 2 \pm 1 \rightarrow \quad 0$$

of the coefficients of

$$\frac{\Delta'}{12} \text{ in } [O] \quad \text{,, } \mp 5 \mp 4 \mp 3 \mp 2 \mp 1 \rightarrow \quad 0$$

$$\frac{1}{6} \frac{\Delta'''}{12^3} \text{ in } [O] \quad \text{,, } \pm 55 \pm 80 \pm 81 \pm 64 \pm 35 \rightarrow \quad 0$$

$$\frac{1}{144} \left(M'' - \frac{317}{144} M^{iv} \right) \text{ in } [E] \quad \text{,, } -11 -20 -27 -32 -35 \rightarrow -36$$

and of the residual coefficients of

$$\frac{1}{2} \frac{M^{iv}}{12^3} \text{ in } [E] \quad \text{viz. } -33 -30 -9 +16 +35 \rightarrow +42$$

6. With regard to these tabulated coefficients it may be noted

(a) That no other interpolation formula gives such simple coefficients as it stands, nor could it be made to do so by any other modification without seriously impairing the simplicity of treatment of the original differences, and also introducing additional sources of cumulative error.

(b) That the second coefficient of $\frac{M''}{144}$ is -20 , i.e. the second coefficient of $\frac{\text{mean } \Delta''}{144}$ is -10 . This is a most happy coincidence,

and it means that the second member of the series of Δ'' corrections happens to be numerically identical with ten times the new middle δ'' , so that we are practically getting two things at once, and we have an opportunity of condensing the corresponding table by making the second column do double duty by acting as vertical argument as well. There are other practical advantages of this coincidence which will appear later. Suffice it here to say

that it is the Δ'' table that has hitherto caused the most trouble in construction and in use, and that it is this very table which is hereby simplified.

(c) That with a view to (i) dispensing as often as possible with Δ^{iv} corrections (as such) without seriously impairing the simplicity of treatment of the original second differences, and (ii) at the same time minimising resulting error (whether individual or mean, neglected or corrected), I have selected the particular device (as in the above table of coefficients) of subtracting $\frac{317}{1728}$ of the mean Δ^{iv} from the mean Δ'' , on the grounds that the resulting "residual Δ^{iv} corrections" are generally negligible, that their mean algebraical value throughout an interval is zero, and their maximum value is about half a thousandth of the original mean Δ^{iv} , so that these residuals may be tabulated in three or four lines only, and then never referred to except as a matter of fact when the Moon is near a maximum declination, and that declination is also a large one by reason of the position of the Moon's node. Other devices might have been chosen, but at the sacrifice of one or more of these practical advantages. Moreover, further pursuit of this same method (such as entrusting the broad fifth-difference effect to the third-differences) would on the whole in each case rather complicate than expedite the work.

7. In the tables, which are purely *additive*, only such information is given as is necessary and sufficient, (a) to produce the end figure of the new quantities, and (b) to enable the calculator to fill in the rest with ease. By this economy the tables are reduced to very small compass.

Comparisons of the Geocentric Places of Uranus, Neptune, and the Sun, calculated from Newcomb's Tables, with their Places calculated from Le Verrier's Tables, for the Year 1904. By A. M. W. Downing, D.Sc., F.R.S.

This paper is a continuation of one printed in *Monthly Notices*, vol. lix., No. 10. The *Nautical Almanac* for 1904 is the first in which Newcomb's Tables of *Uranus* and *Neptune* are used, and a comparison is made at convenient intervals throughout the year between the places there given for Greenwich noon and the corresponding places computed from those given in the *Connaissance des Temps* for 1904 for Paris noon.

I have also added a comparison of the Sun's places, deduced in the same manner, at intervals of eight days throughout the year.

URANUS, 1904.—*Corrections to Le Verrier's Tables.*

Day. 1904.	R.A.		Dec.	Day. 1904.	R.A.		Dec.
	Time. s	Arc.			Time. s	Arc.	
Jan. 1	-33	-45	+0.1	July 3	-34	-47	0.0
Feb. 2	-34	-47	0.0	Aug. 4	-33	-45	+0.2
Mar. 1	-36	-49	0.0	Sept. 1	-34	-47	+0.1
Apr. 2	-36	-49	0.0	Oct. 3	-31	-42	+0.1
May 4	-37	-51	+0.1	Nov. 4	-31	-42	-0.1
June 1	-36	-49	-0.1	Dec. 2	-32	-44	-0.2

NEPTUNE, 1904.—*Corrections to Le Verrier's Tables.*

Day. 1904.	R.A.		Dec.	Day. 1904.	R.A.		Dec.
	Time. s	Arc.			Time. s	Arc.	
Jan. 1	-38	-53	-2.0	July 3	-38	-53	-1.8
Feb. 2	-41	-57	-1.8	Aug. 4	-38	-53	-1.7
Mar. 1	-41	-57	-1.9	Sept. 1	-40	-56	-1.7
Apr. 2	-39	-54	-1.9	Oct. 3	-41	-57	-1.8
May 4	-39	-54	-1.8	Nov. 4	-41	-57	-1.9
June 1	-38	-53	-1.7	Dec. 2	-41	-57	-1.9

SUN, 1904.—*Corrections to Le Verrier's Tables.*

Day. 1904.	R.A.		Dec.	Day. 1904.	R.A.		Dec.
	Time. s	Arc.			Time. s	Arc.	
Jan. 1	-06	-08	-0.2	May 24	-02	-03	+0.1
9	-04	-06	-0.3	June 1	-01	-01	+0.3
17	-04	-06	-0.2	9	-01	-01	+0.2
25	-04	-06	-0.3	17	-02	-03	+0.3
Feb. 2	-03	-04	-0.2	25	-02	-03	+0.2
10	-03	-04	-0.3	July 3	-02	-03	+0.2
18	-03	-04	-0.4	11	-03	-04	+0.3
26	-03	-04	-0.3	19	-02	-03	+0.3
Mar. 5	-03	-04	-0.1	27	-02	-03	+0.3
13	-02	-03	-0.1	Aug. 4	-02	-03	+0.2
21	-03	-05	-0.2	12	-02	-03	+0.2
29	-03	-05	-0.1	20	-03	-04	+0.3
Apr. 6	-01	-01	0.0	28	-02	-03	+0.2
14	-02	-03	-0.1	Sept. 5	-03	-04	+0.2
22	-02	-03	0.0	13	-04	-06	+0.3
30	-01	-01	+0.1	21	-04	-06	+0.3
May 8	-02	-03	+0.2	29	-04	-06	+0.1
16	-01	-01	+0.2	Oct. 7	-05	-07	+0.2

Day. 1904.	R.A.		Dec.	Day. 1904.	R.A.		Dec.
	Time. s	Arc. "	"		Time. s	Arc. "	"
Oct. 15	-05	-07	+03	Nov. 24	-06	-08	-01
23	-06	-09	+01	Dec. 2	-05	-07	-01
31	-06	-09	00	10	-06	-08	-01
Nov. 8	-06	-09	00	18	-07	-10	-02
16	-07	-10	+01	26	-05	-07	-03

Experimental Reduction of some Photographs of Eros made at the Cambridge Observatory for the Determination of the Solar Parallax. By Arthur R. Hinks, M.A.

1. The long series of photographs of the planet *Eros* which I obtained last winter with the new Sheepshanks equatorial of the Cambridge Observatory was very much interrupted by bad weather. My programme was to make four exposures on the planet every hour between 6^h E. and W.—the construction of the instrument precludes greater hour-angles—and, of each set of four, two were made following the stars and letting the planet trail, two following the planet and letting the stars trail. There was only one case of two consecutive nights clear, or nearly so, right through, viz. 1900 November 9 and 10, and the plates taken on these nights, when the planet was but little past opposition, and the parallax factor had become fairly large, were clearly indicated as the subject of preliminary and experimental reductions. The plates were to be centered for each night on the position of the planet at Berlin midnight, the epoch of Professor Millosevich's ephemeris; but, by an accident very fortunate for our present purpose, the plates on the 9th were set to a centering very near that proper to the 10th, and the mistake was discovered in time to use the same point for the latter plates, so that all the exposures to be here treated have approximately the same centering on the sky.

2. As many as twenty exposures were made on one plate, the displacement necessary between each being given to the plate-carrier on rectangular slides on the breach-piece of the telescope, not by displacement of the pointing of the telescope on the sky. There is an obvious advantage in this, that the centre of projection remains unchanged, and all the exposures can be made comparable by linear formulæ of reduction, which is not strictly the case when the pointing of the telescope is changed.

3. The stars on the plates whose meridian places are known are not very symmetrically distributed about the locus of the planet, and many of them are inconveniently bright. It was necessary to consider the possibility of a systematic error due to the trail of either stars or planet—the motion of the planet was

more than a second of arc per minute—and if this exists it is almost necessarily a function of the magnitude. I selected, therefore, fourteen comparison stars well distributed about the locus of the planet, and of magnitudes as nearly as possible equal to it. These fourteen stars were measured with the planet on each exposure, with the new measuring machine which I have recently described (*Monthly Notices*, 1901 May). I may take this opportunity of saying that in accuracy, rapidity, and pleasure of use the machine has more than come up to expectation.

4. *Search for a systematic error due to trail.*—Each set of four exposures was arranged thus :—

$a \dots 3^m$, $b \dots 2^m$, stars fixed ; $c \dots 3^m$, $d \dots 2^m$, planet fixed.

A preliminary comparison showed that for any exposure the differences $b-a$ and $d-c$ for the fourteen comparison stars varied only accidentally from a constant; they could not be represented by a linear function of the coordinates of the stars; that is to say, during the short intervals the variations in differential refraction, orientation, &c., were insensible. In all that follows I have used the means of pairs, $\frac{1}{2}(a+b)$ and $\frac{1}{2}(c+d)$, as single exposures.

Next, an examination of the differences $\frac{1}{2}c + \bar{d} - \frac{1}{2}a + \bar{b}$ gave a similar result; they could not be represented by a linear expression. But they ought to show an effect of error due to trail varying with the magnitude—if this error exists, indeed, at all. To examine this point, four fainter stars, whose images were barely measurable, were added to the above list of fourteen and measured on all the plates. For each of thirteen sets of four exposures the differences $\frac{1}{2}c + \bar{d} - \frac{1}{2}a + \bar{b}$ were formed, their mean taken, and the values of mean—individual found. Then, taking the mean of the thirteen results for each star, we have :

	B.D.	Mean Residual.		B.D.	Mean Residual.
1	+54 434	-2	10	+54 449	+1
2	54 435	-3	11	53 444	+1
3	54 438	-1	12	54 451	-1
4	54 439	-3	13†	54 454	-1
5	54 442	-1	14†	Anon.	+11
6†	54 443	-2	15	Anon.	-3
7	Anon.	0	16	54 459	-2
8	54 445	-3	17†	53 449	+4
9	53 441	-1	18	54 461	-1

It does not seem worth while to add the visual magnitudes. The four stars, photographically fainter, which were added to the original fourteen are distinguished by a dagger. The

quantities are in ten-thousandths of a réseau interval, so that $1=0''\cdot017$. There is evidently something peculiar about star No. 14; it may be a close double. Its large positive residuals have made nearly all the others negative. For the three other faint stars the mean residuals are $-2, -1, +4$. It seems to me that there is no evidence of error depending upon trail varying with the magnitude; that it would certainly do so if it existed at all; and that it is unlikely therefore that it exists as a constant error. I propose, however, to keep the two series—A, stars fixed, B, planet fixed—separate throughout the work. The four faint stars, 6, 13, 14, 17, have not been further employed.

5. *Probable error of a measure.*—Incidentally we may derive from this discussion, if we assume that the differences $b-a, d-c$, and $\frac{1}{2}d+c-\frac{1}{2}b+a$ are entirely accidental and free from the systematic error depending on trail, an estimate of the P.E. of a measure on a single-star disc, *including the real error of position of the apparent centre of the photographic image itself*. Each measure includes two settings on the star disc in each of two positions of the plate differing in orientation by 180° . The results are:—

$$\begin{array}{l} \text{P.E. of a complete measure in } \xi \pm^R \cdot 00053 \pm^R \cdot 00065 \pm^R \cdot 00060 \\ \text{P.E. of a complete measure in } \eta \pm \cdot 00048 \pm \cdot 00052 \pm \cdot 00043. \end{array}$$

$0^R \cdot 0001 = 0''\cdot0175$. We may take as the P.E. of a measure of one image in $\xi \pm 0''\cdot10$, in $\eta \pm 0''\cdot08$.

6. *Reduction of all the plates to standard.*—The fourteen symmetrically arranged comparison stars were selected to provide a strong means of comparing the plates *inter se*. Besides the symmetry of their arrangement, they had the advantage of near equality in magnitude to the planet. Two “zero” exposures were chosen, one for each series, A and B, taken near the meridian on November 10, and all the others were first made comparable with these zero exposures. Following Turner’s method of comparison by linear formulæ, equations of condition were formed of the usual pattern:

$$a\xi + b\eta + c + \xi - \xi' = 0$$

$$d\xi + e\eta + f + \eta - \eta' = 0$$

and for each exposure the six constants were found to reduce it to compare with a zero exposure.

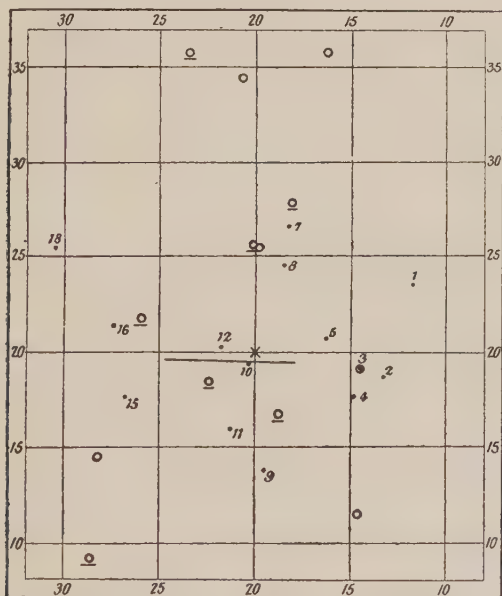
I have preferred to put aside for the present the well-disputed question whether the unsymmetrical part of the differential refraction should be calculated and applied numerically, and the six constants thereby reduced to four. The older method is shorter, and has some advantages in this first experimental work.

We have now each series of exposures comparable among

themselves, but affected still with the differential refraction and aberration, errors of orientation, scale-value, and centering of the zero exposures. On these two exposures only I have measured the images of all the stars on the plate which were to be found in the Harvard A. G. Zone Catalogue—there were scarcely enough stars in the Paris list, nor were they well distributed (see figure). The rectangular coordinates of these standard stars were computed for 1900.0 with assumed centre of plate

$$\alpha \ 1^{\text{h}} 57^{\text{m}} 8^{\text{s}} \qquad \delta + 54^{\circ} 22' 0''$$

and the six constants to reduce each zero exposure to standard were found as above.



Centre of plate (1900.0) $\alpha \ 1^{\text{h}} 57^{\text{m}} 8^{\text{s}} \ \delta + 54^{\circ} 22' 0''$.

• Comparison star. — Path of planet.

o Standard star; underlined o if in Paris list.

Half scale. Every fifth R. line only shown. $5R. = 14'.6$.

The reduction of all the exposures to standard for 1900.0 proceeded, therefore, in two steps:

(1) Each exposure was reduced to the zero of its series by the aid of constants derived from the fourteen comparison stars.

(2) All were then reduced from zero to standard by the constants derived from the thirteen meridian standard stars (of which only two had served also as comparison stars).

The advantages of this double procedure seem to me to be as

follows :—In addition to the fact that the comparison stars are well distributed, and nearly equal to the planet in magnitude, we have the further advantage that there is no question of meridian places in the first step. The exposures are made comparable with one another by a process that may be considered final. No revision of meridian places can affect it. On the other hand, the reduction from zero to standard depends on the accuracy of the coordinates of the standard stars calculated from their meridian places; it may well have to be revised when better places are available. When this is done there will be only two sets of equations to solve again, instead of twenty-six.

I do not, however, anticipate that any revision of meridian places will seriously alter my constants for the reduction to standard, except perhaps the errors of centering c and f , which will be affected by the systematic difference between the fundamental systems of Auwers and Newcomb. The average residuals in the reduction of the thirteen standard stars are in ξ $0''.52$ and in η $0''.44$, which speaks well for the original accuracy of the Harvard places, affected as they are now by unknown proper motions for about twenty-five years.

7. *Search for evidence of the effect of atmospheric dispersion.*—The possibility of systematic error due to atmospheric dispersion is an objection that will be alleged against the determination of the solar parallax by photography. To this may be added in the present case the possibility of variable distortion due to flexure of the plane mirror of the Cambridge equatorial varying with the hour-angle. The two effects may here be included in the category "hour-angle error" as used by Kapteyn. It is possible to find in the reduction of all the exposures to a zero near the meridian a little evidence on this matter. In series A I have taken six exposures, three on each side of the meridian, and formed for each star the differences of the reduced values of ξ from the values on the zero exposure. They are given in the following table, the unit as before being the ten-thousandth of a réseau interval.

Hour-Angle.	East.			West.			$\frac{1}{3}(\Sigma W - \Sigma E).$
	$-4^h 9.$	$-4^h 0.$	$-2^h 9.$	$+2^h 9.$	$+4^h 0.$	$+4^h 9.$	
Star 1	0	- 5	- 3	+ 2	0	+ 3	+ 4
2	+ 5	- 2	- 9	- 5	+ 3	+ 2	+ 3
3	- 4	+ 6	+ 2	+ 10	+ 4	+ 1	+ 4
4	- 1	+ 4	+ 7	+ 2	- 4	+ 3	- 3
5	+ 2	+ 1	+ 4	+ 5	+ 2	+ 6	+ 2
7	- 12	- 11	- 13	- 11	- 12	- 14	0
8	+ 2	+ 1	0	- 2	- 3	- 2	- 3
9	+ 8	+ 6	+ 11	0	+ 9	0	- 5
10	- 6	- 11	- 11	+ 5	- 4	+ 2	+ 10
11	+ 4	0	+ 2	+ 1	+ 2	+ 12	+ 3

Hour-Angle.	East.				West.		$\frac{1}{3}(\Sigma W - \Sigma E)$.
	$-4^h 9.$	$-4^h 0.$	$-2^h 9.$	$+2^h 9.$	$+4^h 0.$	$+4^h 9.$	
Star 12	+ 1	- 7	+ 4	- 1	+ 1	- 7	- 2
15	- 5	+ 4	- 7	- 11	- 11	- 13	- 9
16	0	- 4	+ 7	+ 7	+ 10	+ 1	+ 5
18	+ 8	+ 17	+ 5	- 1	+ 2	+ 5	- 8

If a star is extra blue its ξ is increased W. and diminished E.; the above residuals are algebraically diminished W. and increased E., so that the value of W.—E. (which is independent of the errors of measurement on the zero plate, except in so far as they affect the constants of reduction) is negative for an extra blue star. Taking the P.E. of a single measure in ξ as 6, the probable value of a quantity in the last column, allowing a little for accumulated errors in the reduction to zero, will be about 4—it must be remembered that each residual represents the mean of two exposures. Of the fourteen quantities in the last column, five are above 4, two equal to it, and seven below 4. There is, therefore, no indication that these quantities are other than purely accidental. The evidence is slight, but it is at least in the right direction.

8. *Reduced measures of the planet.*—The following table contains the mean epoch of exposure, expressed in decimals of a day from Berlin mean noon—called afterwards $T + \Delta T$ —the measured coordinates of the planet (means of a and b or of c and d), the same reduced to zero, and then to standard, and finally these last referred to the centre of the plate (20.0 20.0) as origin, and multiplied by the constant .00085221 to express them in terms of the radius of projection.

TABLE I.
Measures of the planet.

Series A.	Measures. ξ and η	Red. to Zero.	Red. to Standard.	In parts of Radius.
d.				
1 Nov. 9.552382	24.8337	24.6146	24.5935	+ .00391463
	19.9144	19.7026	19.6764	- .00027578
2 .605387	24.4737	24.2663	24.2453	+ .00361789
	19.9812	19.6846	19.6564	- .00029282
3 .649279	24.2036	23.9821	23.9612	+ .00337577
	20.1298	19.6663	19.6365	- .00030978
4 .688555	23.9549	23.7352	23.7144	+ .00316545
	20.1703	19.6461	19.6149	- .0003281
5 .729413	23.7058	23.4828	23.4621	+ .00295044
	20.2200	19.6207	19.5880	- .00035111
6 10.277103	20.2625	20.2907	20.2704	+ .00023044
	19.8577	19.5107	19.4596	- .00046053

Series A.		Measures. ξ and η	Red. to Zero.	Red. to Standard.	In parts of Radius.
d.					
7	Nov. 10'317877	20'0017	20'0315	20'0112	+ '00000954
		19'7320	19'5137	19'4612	- '00045917
8	'356861	19'7636	19'7814	19'7611	- '00020359
		19'6065	19'5121	19'4581	- '00046181
9	'464122	19'0770	zero	19'0568	- '00080380
		19'4933	exposure	19'4353	- '00048124
10	'515599	18'9717	18'7358	18'7157	- '00109449
		19'4730	19'4724	19'4124	- '00050076
11	'565637	18'6214	18'4073	18'3873	- '00137436
		19'5340	19'4453	19'3834	- '00052547
12	'605862	18'3782	18'1460	18'1261	- '00159696
		19'6375	19'4182	19'3548	- '00054976
13	'650159	18'0889	17'8614	17'8417	- '00183932
		19'7751	19'3849	19'3199	- '00057959
Series B.					
1	Nov. 9'556968	24'6744	24'4636	24'5632	+ '00388880
		19'9127	19'6972	19'6746	- '00027730
2	'609456	24'3153	24'1187	24'2184	+ '00359496
		19'9880	19'6793	19'6547	- '00029427
3	'653376	24'0454	23'8368	23'9366	+ '00335481
		20'1384	19'6605	19'6343	- '00031165
4	'693284	23'7892	23'5845	23'6844	+ '00313988
		20'1689	19'6388	19'6112	- '00033134
5	'733597	23'5449	23'3372	23'4372	+ '00292922
		20'2229	19'6170	19'5879	- '00035120
6	10'281374	20'1128	20'1415	20'2420	+ '00020623
		19'8306	19'5071	19'4598	- '00046036
7	'322405	19'8471	19'8811	19'9815	- '00001577
		19'7276	19'5089	19'4601	- '00046011
8	'361507	19'6090	19'6310	19'7314	- '00022890
		19'6030	19'5080	19'4577	- '00046215
9	'468533	18'9275	zero	19'0280	- '00082835
		19'4876	exposure	19'4333	- '00048295
10	'519523	18'8336	18'5904	18'6910	- '00111554
		19'4762	19'4669	19'4107	- '00050221
11	'569935	18'4646	18'2580	18'3588	- '00139865
		19'5361	19'4384	19'3803	- '00052811
12	'610161	18'2281	17'9975	18'0984	- '00162056
		19'6397	19'4104	19'3508	- '00055325
13	'654831	17'9314	17'7113	17'8124	- '00186429
		19'7830	19'3769	19'3157	- '00058317

9. *Comparison with the ephemeris. General considerations.*—

We are now in a position to compare with the ephemeris, and here we come to the essential point of what I have called this experimental reduction. It was suggested by Mr. Cowell in *The Observatory* (1900 December) that there are clear advantages in using in astrographic methods a heliocentric ephemeris of the planet expressed in rectangular coordinates. He points out that the reduction of the measured positions of stars and planet to standard, by the linear transformations, corrects both stars and planet completely for the aberration of light (excepting a small term depending on the eccentricity of the Earth's orbit). For the planet this is over-correction, inasmuch as the usual method of allowing for the motion of the planet during the time the light is on its way from it to us, is to leave it uncorrected for the aberration of light and to antedate the observation by the light-time. We thus obtain the true position of the planet at the antedated time, freed from both planetary aberration and the aberration of light. If, however, we correct automatically the planet with the stars for the aberration of light, we obtain the position of the planet at the time the light left it as seen from the Earth at the time the light reached it; and this is not immediately comparable with the ephemeris in its ordinary form. This difficulty is evaded if we proceed in the following manner, for the idea of which I must acknowledge my indebtedness to Mr. Cowell's paper.

10. Let the unit of distance be the mean distance of the Earth from the Sun, and let U, V, W be the heliocentric coordinates of the planet at the epoch T (the time the light left it) referred to three rectangular axes, S_u, S_v, S_w . S is the centre of the Sun; the positive directions of the axes are S_u towards the vernal equinox for 1900.0; S_v in the plane of the mean ecliptic for this epoch, at right angles to S_u , towards Cancer; S_w towards the north pole of the ecliptic.

Now let f, g, h be the heliocentric coordinates referred to the same axes of the centre of the Earth at epoch $T + \Delta T$ (the time when the light which left the planet at T reaches the Earth). Transform to parallel axes through the centre of the Earth, E .

$U - f, V - g, W - h$, are the coordinates of the planet at epoch T as seen from the centre of the Earth E at epoch $T + \Delta T$. (System I.)

Now transform to a new set of axes through the same origin E, x, y, z ; Ez is drawn through the point on the celestial sphere corresponding to the assumed centre of the plate for 1900.0; Ey is at right angles to this in the plane of the meridian of the plate; Ex is at right angles to the others. Ex, Ey are then parallel to the axes of coordinates on the standard plate. (System II.)

If α, δ is the assumed centre of the plate, the direction cosines of the axes of System II. referred to the axes of System I. are easily found to be

$$\begin{aligned}
l_1 &= -\sin \alpha & m_1 &= -\sin \delta \cos \alpha & n_1 &= \cos \delta \cos \alpha \\
l_2 &= \cos \epsilon \cos \alpha & m_2 &= \sin \epsilon \cos \delta & n_2 &= \sin \epsilon \sin \delta \\
& & & -\cos \epsilon \sin \delta \sin \alpha & & +\cos \epsilon \cos \delta \sin \alpha \\
l_3 &= -\sin \epsilon \cos \alpha & m_3 &= \cos \epsilon \cos \delta & n_3 &= \cos \epsilon \sin \delta \\
& & & +\sin \epsilon \sin \delta \sin \alpha & & -\sin \epsilon \cos \delta \sin \alpha.
\end{aligned}$$

And with respect to the axes of System II. the coordinates of the planet at epoch T, as seen from E at epoch T + ΔT , are

$$\begin{aligned}
x &= l_1 (U-f) + l_2 (V-g) + l_3 (W-h) = L \\
y &= m_1 (U-f) + m_2 (V-g) + m_3 (W-h) = M \\
z &= n_1 (U-f) + n_2 (V-g) + n_3 (W-h) = N
\end{aligned}$$

Again, let x_o, y_o, z_o be the coordinates of the Observatory O at epoch T + ΔT referred to the axes of System II. It is easily seen that in terms of the mean distance R of Earth from Sun

$$\begin{aligned}
x_o &= \frac{\rho}{R} \cdot \frac{\rho'}{\rho} \cos \phi' \sin h & &= \pi a \\
y_o &= \frac{\rho}{R} \cdot \frac{\rho'}{\rho} (\sin \phi' \cos \delta - \cos \phi' \sin \delta \cos h) & &= \pi b \\
z_o &= \frac{\rho}{R} \cdot \frac{\rho'}{\rho} (\sin \delta \sin \phi' + \cos \delta \cos \phi' \cos h) & &= \pi c
\end{aligned}$$

where h is the hour-angle of the centre of the plate, ϕ' the reduced latitude of the Observatory, ρ' and ρ the local and equatorial radii of the Earth, π the Sun's equatorial horizontal parallax.

Transform finally to another set of axes through O, parallel to the axes of System II.

$$\begin{aligned}
\text{Then} \quad X &= L - a\pi \\
Y &= M - b\pi \\
Z &= N - c\pi
\end{aligned}$$

are the coordinates of the planet at epoch T as seen from the Observatory at epoch T + ΔT .

And the position of the planet projected upon the ideal standard plate will be

$$\begin{aligned}
\xi_o &= \frac{X}{Z} = \frac{1}{N} \left\{ L - \left(a - \frac{c}{N} \right) \pi \right\} = \nu \{ L - (a - c) \pi \} \\
\eta_o &= \frac{Y}{Z} = \frac{1}{N} \left\{ M - \left(b - \frac{c}{N} \right) \pi \right\} = \nu \{ M - (b - c) \pi \}
\end{aligned}$$

since πc is small compared with N; ν is written for $1/N$ to avoid fractions in the printing.

Now in forming the equations of condition we must put $\pi = \pi_0 + \Delta\pi$, and it will be necessary to introduce tabular corrections to the ephemeris position of the planet—it will be assumed for the present that the heliocentric ephemeris of the Earth is sufficiently good. If these corrections are $\Delta U, \Delta V, \Delta W$, we have for the true ξ coordinate of the planet

$$\frac{L + l_1 \Delta U + l_2 \Delta V + l_3 \Delta W - a(\pi_0 + \Delta\pi)}{N + n_1 \Delta U + n_2 \Delta V + n_3 \Delta W - c(\pi_0 + \Delta\pi)} \\ = \nu \{L - \pi_0(a - \nu c)\} + \nu(l_1 \Delta U + l_2 \Delta V + l_3 \Delta W) - \nu(a - c\nu)\Delta\pi$$

which we will write

$$\xi = \xi_0 + \Delta\xi_0 - \nu(a - c\nu)\Delta\pi$$

and similarly

$$\eta = \eta_0 + \Delta\eta_0 - \nu(b - c\nu)\Delta\pi$$

Hence each reduced measure (ξ, η) of the planet gives us two equations of condition—

$$\begin{aligned} \nu(a - c\nu)\Delta\pi - \Delta\xi_0 + \xi - \xi_0 &= 0 \\ \nu(b - c\nu)\Delta\pi - \Delta\eta_0 + \eta - \eta_0 &= 0 \end{aligned}$$

11. *The ephemerides.*—With very great kindness Professor Millosevich computed for me a heliocentric ephemeris of *Eros* for a part of the period during which it was under observation. My sincere acknowledgments are due to him for the readiness with which he undertook this indispensable part of the experiment.

This ephemeris, he writes, is founded upon the following elements :—

Osculation 1900 Oct. 31.5 B = epoch.

M	304° 24' 49".75
μ	2015".23858
ϕ	12° 52' 49".93
ω	177° 39' 8".65
Ω	303° 30' 42".38
i	10° 49' 38".03
$\log a$	0.1637867.

These elements represent, account being taken of perturbations by *Venus, Earth, Mars, Jupiter, and Saturn*, within one second of arc, and generally within much less than that, all the normal places available between 1898 August and 1901 April.

Heliocentric ephemeris of Eros referred to the ecliptic of 1900.0.
Professor E. Millosevich.

12 ^h Berlin M.T. 1900.		Hel. Longitude.	Hel. Latitude.	Log. Rad. vect.
Oct.	19	33° 44' 48" 31	+ 10° 49' 37" 72	0.1373172
	20	34 22 33.08	49 33.71	62860
	21	35 0 28.64	49 25.06	52529
	22	35 38 35.02	49 11.70	42182
	23	36 16 52.26	48 53.58	31819
	24	36 55 20.39	48 30.62	21442
	25	37 33 59.48	48 2.76	11051
	26	38 12 49.53	47 29.93	0.1300647
	27	38 51 50.73	46 52.07	0.1290232
	28	39 31 2.94	46 9.10	79808
	29	40 10 26.27	45 20.98	69376
	30	40 50 0.73	44 27.62	58938
	31	41 29 46.52	43 28.97	48495
Nov.	1	42 9 43.47	42 24.95	38049
	2	42 49 51.68	41 15.50	27600
	3	43 30 11.20	40 0.55	17150
	4	44 10 42.04	38 40.05	0.1206701
	5	44 51 24.20	37 13.95	0.1196254
	6	45 32 17.74	35 42.15	85811
	7	46 13 22.71	34 4.56	75374
	8	46 54 39.11	32 21.14	64945
	9	47 36 6.93	30 31.86	54525
	10	48 17 46.20	28 36.64	44117
	11	48 59 36.96	26 35.41	33721
	12	49 41 39.19	24 28.11	23339
	13	50 23 52.89	22 14.68	12974
	14	51 6 18.08	19 55.07	0.1102627
	15	51 48 54.78	17 29.22	0.1092301
	16	52 31 42.99	14 57.07	81998
	17	53 14 42.70	12 18.56	71719
	18	53 57 53.91	9 33.62	61466
	19	54 41 16.64	6 42.22	51242
	20	55 24 50.90	+ 10 3 44.28	0.1041048

The heliocentric ephemeris of the Earth is taken from the *Berliner Jahrbuch*, 1900.

Both these ephemerides are for the mean equinox 1900.0 of Leverrier's tables.

From them the ephemerides of the planet and the Earth expressed in rectangular coordinates referred to the first set of axes, through the centre of the Sun, were constructed with seven-figure logarithm tables.

12. *Formation of the equations of condition.*—The mean epochs of the exposures $T + \Delta T$ reduced to Berlin M.T. are given in Table I.

The values of ΔT , the light equation, were calculated from the values of $\log \Delta$ given by Millosevich in *A.N.* No. 3660, 1, using Gill's value of the Sun's light equation $498^s.46$. Thus T was found.

With these values of $T - 0^d.5$ (since the ephemeris of the planet is for midnight) and of $T + \Delta T$, the values of U, V, W, f, g, h were interpolated. The third differences are small and somewhat irregular, owing largely, no doubt, to the use of only seven-figure tables.

The values of the nine direction cosines required (see paragraph 10) are :—

$l_1 - .4891288$	$m_1 - .7089005$	$n_1 + .5081469$
$l_2 + .8001604$	$m_2 - .1328410$	$n_2 + .5848907$
$l_3 - .3471266$	$m_3 + .6926858$	$n_3 + .6322104$

The values of a, b, c were formed with the data for the Cambridge Observatory

$$\phi' = 52^\circ 1' 29''.2$$

$$\log \rho' / \rho = \bar{1} .999078$$

It should be remarked that from this point no more logarithms were used. The whole of the rest of the calculations, as well as all the foregoing interpolations, were done on an arithmometer (the Thomas de Colmar), which calculates quantities of the form

$$l_1(U-f) + l_2(V-g) + l_3(W-h)$$

with great ease. It seems to me that this possibility of using an arithmometer almost throughout is a strong point of the method.

Table II. contains the quantities required in the formation of the equations of condition.

								TA
								Quantities required in the format
Series A.								
No.	T-0 ⁴ .5	U	V	W	f	g	h	
1	Nov. 9'050204 +	8641836 +	9476201 +	2378653 +	6731748 +	7258077 +	5 × 1	
2	103210	34709	80585	7979	24912	64246	5	
3	147103	28805	84212	7421	19248	69348	5	
4	186380	23520	87456	6920	14175	73911	6	
5	227238	8618020	9490828	2376399	6708895	7278654	6	
6	774938	8544107	9535828	2369364	6637790	7341874	9	
7	815713	38591	39163	8837	32472	46554	9	
8	854698	33315	42349	8332	27384	51025	9	
9	9'961960	18790	51106	6940	13370	63309	9	
10	10'013438	11814	55303	6271	06636	69196	10	
11	063477	8505030	59380	5620	6600085	74912	10	
12	103702	8499575	62655	5096	6594815	79503	10	
13	10'147999 +	8493565 +	9566259 +	2364518 +	6589008 +	7384555 +	10	
Series B.								
1	9'054790 +	8641220 +	9476581 +	2378595 +	6731157 +	7258612 +	5 × 1	
2	107279	34162	80921	7927	24387	64719	5	
3	151200	28254	84551	7368	18719	69825	5	
4	191109	22883	87846	6860	13564	74460	6	
5	231422	8617456	9491173	2376346	6708354	7279139	6	
6	779209	8543529	9536177	2369309	6637233	7342364	9	
7	820241	37978	39533	8778	31881	47073	9	
8	859344	32686	42729	8272	26777	51558	9	
9	9'966371	18192	51466	6883	12793	63814	9	
10	10'017362	11282	55623	6220	6606122	69644	10	
11	067775	8504447	59730	5564	6599522	75403	10	
12	108001	8498992	63005	5040	94252	79993	10	
13	10'152671 +	8492931 +	9566639 +	2364457 +	6588395 +	7385087 +	10	

The equations of condition.

L	M	N	$\nu(a-c\nu)$	$\nu(b-c\nu)$	ξ_0	η_0
00148839	-00010645	+37717714	+06742	+00153	+00391740	-00028288
138320	10879	701534	11249	02662	362088	29990
129628	11084	688159	14031	05539	339970	31770
121853	11306	676194	15613	08552	316769	33652
113768	11534	663763	+16248	11916	295138	35702
00006144	16176	498432	-15695	08430	023073	46730
00001820	16636	486228	13991	05318	+00001107	46645
009430	17094	474564	11500	+02762	-00020263	46792
030320	18428	442538	-01661	-01005	080269	48789
040333	19109	427188	+03633	-00748	109312	50737
050048	19789	412302	08427	+00802	137365	53236
057855	20359	400352	11700	02879	159677	55662
00066442	-00021000	+37387197	+14439	+05851	-00183866	-00058662
00147923	-00010663	+37716314	+07200	+00333	+00389131	-00028414
137511	10901	700292	11548	02901	359827	30151
128815	11111	686905	14236	05836	335736	31969
120917	11326	674752	15742	08932	314242	33869
112944	11552	662492	+16247	12498	292960	35998
00005310	16220	497153	-15564	08087	+00020794	46703
00002700	16690	484871	13743	04996	-00001346	46654
010338	17150	473178	11156	+02493	022834	46828
031180	18482	441223	-01209	-01037	082762	48920
041092	19164	426025	+04026	-00671	111511	50919
050884	19848	411021	08804	+00989	139765	53475
058683	20419	399077	12008	03139	162027	55936
00067343	-00021072	+37385815	+14664	+06201	-00186379	-00059007

Equations of condition.

Series A.			n_1	n_2	v
ξ	1	$+0.67\Delta\pi - 1\Delta\xi_0$	-28×10^{-7}	$-2 \times 10^{-7} = 0$	$+4 \times 10^{-7}$
	2	$+1.12 - 1$	-30	-6	-1
	3	$+1.40 - 1$	-39	-16	-9
	4	$+1.56 - 1$	-22	0	+5
	5	$+1.62 - 1$	-9	+12	+17
	6	$-1.57 - 1$	-3	+3	+9
	7	$-1.40 - 1$	-15	-10	-4
	8	$-1.15 - 1$	-10	-6	0
	9	$-0.17 - 1$	-11	-10	-4
	10	$+0.36 - 1$	-14	-14	-8
	11	$+0.84 - 1$	-7	-9	-3
	12	$+1.17 - 1$	-2	-5	0
	13	$+1.44 - 1$	-7	-11	-6
η	1	$+0.02 - 1\Delta\eta_0$	+71	+67	-1
	2	$+0.27 - 1$	+71	+67	-1
	3	$+0.55 - 1$	+79	+76	+8
	4	$+0.86 - 1$	+83	+80	+12
	5	$+1.19 - 1$	+59	+56	-12
	6	$+0.84 - 1$	+68	+67	-1
	7	$+0.53 - 1$	+73	+72	+4
	8	$+0.28 - 1$	+61	+60	-8
	9	$-0.10 - 1$	+67	+67	-1
	10	$-0.07 - 1$	+66	+66	-2
	11	$+0.08 - 1$	+69	+69	+1
	12	$+0.29 - 1$	+69	+69	+1
	13	$+0.59 - 1$	+70	+71	+3
Series B.			n_1	n_2	v
ξ	1	$+0.72\Delta\pi - 1\Delta\xi_0$	-25×10^{-7}	$-4 \times 10^{-7} = 0$	$+2 \times 10^{-7}$
	2	$+1.15 - 1$	-33	-13	-9
	3	$+1.42 - 1$	-26	-7	-3
	4	$+1.57 - 1$	-25	-7	-4
	5	$+1.62 - 1$	-4	+13	+16
	6	$-1.56 - 1$	-17	-12	+2
	7	$-1.37 - 1$	-23	-19	-6
	8	$-1.12 - 1$	-6	-3	+9
	9	$-0.12 - 1$	-7	-6	+3
	10	$+0.40 - 1$	-4	-4	+3

Series B.			n_1	n_2	v
11	$+0.88\Delta\pi - 1\Delta\xi_0$		-10×10^{-7}	$-12 \times 10^{-7} = 0$	-7×10^{-7}
12	$+1.20$	-1	-3	-5	-1
13	$+1.47$	-1	-5	-8	-5
η 1	$+0.03$	$-1\Delta\eta_0$	$+68$	$+60$	-5
2	$+0.29$	-1	$+72$	$+65$	-1
3	$+0.58$	-1	$+80$	$+71$	$+4$
4	$+0.89$	-1	$+74$	$+66$	-2
5	$+1.25$	-1	$+88$	$+82$	$+13$
6	$+0.81$	-1	$+67$	$+65$	-3
7	$+0.50$	-1	$+64$	$+63$	-4
8	$+0.25$	-1	$+61$	$+60$	-6
9	-0.10	-1	$+62$	$+62$	-3
10	-0.07	-1	$+70$	$+70$	$+5$
11	$+0.10$	-1	$+66$	$+67$	$+2$
12	$+0.31$	-1	$+61$	$+62$	-4
13	$+0.62$	-1	$+69$	$+70$	$+3$

The numerical terms which arise from the combination of the quantities of Table II. are found in the column n_1 . They are expressed in terms of the length of the radius of projection on to the plate; that is to say, they are independent of the focal length of the particular telescope employed, so that the equations could be combined immediately with the work of other telescopes reduced in the same way.

13. *Empirical correction of the ephemeris.*—A glance at the terms in column n_1 of the ξ equations shows that the equations cannot be satisfied as they stand. We must introduce a correction to the ephemeris varying with the time.

An approximate value of this correction was found as follows:

Taking groups of equations of condition which have similar parallax factors on the two nights, and writing them in the form

$$a\Delta\pi + \Delta\xi_0 + \Delta'\xi_0 t + n = 0$$

we have from among the ξ equations of Series A:

1	$+0.67$	$\Delta\pi + \Delta\xi_0$	$+0.05$	$\Delta'\xi_0$	$-28=0$
2	$+1.12$		$+0.10$		-30
3	$+1.40$		$+0.15$		-39
11	$+0.84$		$+1.06$		-7
12	$+1.17$		$+1.10$		-2
13	$+1.44$		$+1.15$		-7

and subtracting the mean of the first three from the mean of the second

$$0.09 \Delta\pi + 1.00 \Delta'\xi_0 + 27 = 0.$$

The effect of $\Delta\pi$ is inappreciable. Proceeding in the same way with the other groups we find that the corrections to be added to the numerical terms to allow for an error in the ephemeris varying with the time are

$$\begin{array}{rcl} \Delta'\xi_0 = & \text{Series A} - 27t & \text{Series B} - 22t \\ \Delta'\eta_0 = & + 4t & + 8t \end{array}$$

Reckoning t from Berlin midnight on November 10, the corrected values of the numerical terms are found in the column n_2 of the equations of condition.

This does not, however, necessarily represent a real correction to the assumed motion of the planet. The greater part may be due to the accumulation of error in adding two separate ephemerides each computed with seven-figure logarithm tables. The necessity for it should disappear with more accurate ephemerides. It has been used here as a temporary expedient, and it is practicable only when we have observations on two consecutive nights at the same hour-angles, which was not a common case last winter.

14. *Solution of the equations of condition.*—In this preliminary solution equal weight has been given to each equation.

The normal equations from the twenty-six equations of condition of Series A are :

$$\begin{array}{rcl} +22.83 \Delta\pi - 5.89 \Delta\xi_0 - 5.33 \Delta\eta_0 + 336.59 & = & 0 \\ - 5.89 \Delta\pi + 13.00 \Delta\xi_0 & 0.00 \Delta\eta_0 + 74.00 & = 0 \\ - 5.33 \Delta\pi & 0.00 \Delta\xi_0 + 13.00 \Delta\eta_0 - 887.00 & = 0 \end{array}$$

And their solution gives

$$\begin{array}{rcl} \text{P.E. of one equation } \pm 4.63 \text{ or} & \pm 0''.096 & \text{Wt.} \\ \Delta\pi = - 0.36 \pm 1.09 \text{ ,, } - 0''.007 \pm 0''.023 & 17.97 \\ \Delta\xi_0 = - 5.85 \pm 1.38 \text{ ,, } - 0.12 \pm 0.028 & 11.32 \\ \Delta\eta_0 = + 68.08 \pm 1.36 \text{ ,, } + 1.40 \pm 0.028 & 11.59 \end{array}$$

Similarly for Series B':

$$\begin{array}{rcl} +23.31 \Delta\pi - 6.26 \Delta\xi_0 - 5.46 \Delta\eta_0 + 381.65 & = & 0 \\ - 6.26 \Delta\pi + 13.00 \Delta\xi_0 & 0.00 \Delta\eta_0 + 87.00 & = 0 \\ - 5.46 \Delta\pi & 0.00 \Delta\xi_0 + 13.00 \Delta\eta_0 - 863.00 & = 0 \end{array}$$

$$\text{P.E. of one equation } \pm 4.26 \text{ or} \quad \pm 0''.088 \quad \text{Wt.}$$

$$\begin{array}{rcl} \Delta\pi = - 3.40 \pm 1.01 \text{ ,, } - 0''.070 \pm 0''.021 & 18.00 \\ \Delta\xi_0 = - 8.33 \pm 1.28 \text{ ,, } - 0.17 \pm 0.026 & 11.13 \\ \Delta\eta_0 = + 64.85 \pm 1.25 \text{ ,, } + 1.34 \pm 0.026 & 11.53 \end{array}$$

The residuals which arise from substituting these values of the unknowns in the equations of condition are given in the last column of the table. It will be noticed that the residuals in the four equations No. 5, which represent the last set of exposures on November 9, are unduly large. I have remeasured the images of the planet and examined the reductions, and can find no error. The zenith distance was about 50° and the images are bad; indeed throughout the whole of these two nights the seeing was not at all good; they were the first frosty nights of the winter.

It does not seem necessary to give the value of π_0 which has been assumed in these reductions, as it is inadvisable to multiply results which are in no sense definitive, even for the small number of observations here discussed.

15. The difference $0''.06$ between the results of Series A (stars fixed) and Series B (planet fixed) looks somewhat large, though it is well within the possible difference of two results each with a P.E. $\pm 0''.02$. For the present I should prefer to look upon it as bad luck, and should deprecate the drawing of any conclusion confirmatory of the fears that have been expressed that systematic error may arise from the motion of the planet. If such error were independent of the magnitude, and a function only of the motion of the planet and the time of exposure—that is to say, of the length of the trail—each series would give a value of the parallax free from systematic error, since the time of exposure and the motion of the planet were sensibly constant for the two nights; and both would give similarly erroneous values of the tabular corrections. If the error were a function of the magnitude, which varies effectively with the zenith distance, it should nevertheless be eliminated by the method here employed of reducing by means of comparison stars very nearly equal to the planet in magnitude. Moreover some evidence has been given that, even for stars on the limit of photographic action for the particular exposure, no certain trace of the effects of trail is discoverable. I prefer then to draw no further conclusion from the discrepancy than that it will be wise to adhere to my resolution to keep the two series separate right through the work.

16. *Tabular errors of the planet.*—It is a not inconsiderable advantage of this method that we can easily derive from the values found for $\Delta\xi_0$, $\Delta\eta_0$, the corrections to the tabular heliocentric longitude and latitude of the planet, if we assume that the ephemeris of the Earth is accurate.

From the relations between U , V , W and λ , β , R , the heliocentric longitude, latitude, and radius vector, we have

$$\begin{aligned}\Delta U &= \cos \beta \cos \lambda \cdot \Delta R - R \cos \beta \sin \lambda \cdot \Delta \lambda - R \cos \lambda \sin \beta \cdot \Delta \beta \\ \Delta V &= \cos \beta \sin \lambda \cdot \Delta R + R \cos \beta \cos \lambda \cdot \Delta \lambda - R \sin \lambda \sin \beta \cdot \Delta \beta \\ \Delta W &= \sin \beta \cdot \Delta R \quad \quad \quad + R \cos \beta \cdot \Delta \beta\end{aligned}$$

And we had

$$\nu \Delta \xi_0 = l_1 \Delta U + l_2 \Delta V + l_3 \Delta W$$

$$\nu \Delta \eta_0 = m_1 \Delta U + m_2 \Delta V + m_3 \Delta W$$

Putting in the values of β , λ , R , l_1 . . . &c. we obtain two equations for $\Delta\lambda$ and $\Delta\beta$ in terms of ΔR :

$$\begin{aligned} 1.148 \Delta\lambda - 0.509 \Delta\beta &= -1.89 - 0.204 \Delta R \\ + 0.564 \Delta\lambda + 1.021 \Delta\beta &= +17.73 + 0.435 \Delta R \end{aligned}$$

whence

$$\Delta\lambda = +4.9 + 0.009 \Delta R$$

$$\Delta\beta = +14.7 + 0.421 \Delta R.$$

ΔR must be derived from an extended comparison with observation of the theory of the planet's motion. Putting it zero for the present, we have $\Delta\lambda = +0''.10$ $\Delta\beta = +0''.30$.

There is, therefore, no need to introduce a geocentric ephemeris of the planet at any point of the work. I have used the values of $\log \Delta$ from Millosevich's ephemeris to get the light equation, because they were ready to hand. But the geocentric distances of the planet are easily found from

$$\Delta^2 = (U - f)^2 + (V - g)^2 + (W - h)^2.$$

17. *Conclusions.*—To sum up, I should suggest that the following conclusions may be drawn from the results of this experimental reduction of a small series of *Eros* plates.

(1) The smallness of the probable error of an equation of condition, even with a comparatively rough ephemeris, is a good omen for the ultimate success of the enterprise.

(2) It is an absolute necessity to have ephemerides computed with eight-figure logarithms. This is indeed a truism, for since Sir David Gill found that it was necessary for his heliometer determination of the solar parallax, we should be wasting time now if our results were not good enough to demand it.

(3) There is only one complete set of eight-figure tables in existence—viz. those published by the French *Service Géographique de l'Armée*, which are based on the centesimal division of the quadrant. *We must therefore work in the centesimal system.* Professor Bauschinger kindly informs me that the ephemeris of Victoria for Sir David Gill was computed in the K. Rechen Institut, Berlin, in grades, and afterwards converted into degrees, “was einige lästige Mehrarbeit verursacht.” There does not seem to be any good reason why we should give ourselves this extra labour.

(4) I hope that the present reduction has sufficiently shown the advantages which arise from working with the separate heliocentric ephemerides of the Earth and *Eros*, instead of with the usual geocentric ephemeris of the planet. Briefly, they are

(a) the ephemeris of the Earth can be computed once for all ; the ephemeris of the planet can with comparatively small labour be improved from time to time as we proceed, and the corrections can be carried into our equations of condition with the minimum of trouble. (b) The interpolations from the heliocentric ephemerides are not so troublesome as from a geocentric ; the quantities vary more regularly. (c) The greater part of the calculation is done on a machine instead of with logarithms, which saves a deal of writing. (d) The form of the work is comparatively simple, and you see clearly what you are doing all through.

(5) Since the sub-committee of the *Comité International Permanent* is publishing the revised places of the standard stars referred to Newcomb's Fundamental Catalogue, it will be necessary to refer the elements of the orbit of *Eros* to Newcomb's system—at present they are based on Leverrier's—and to compute the ephemeris of the Earth from Newcomb's tables of the Sun, with Gill's value for the mass of the Moon.

(6) Finally, the great advantage is that photographic results reduced on these lines by different observatories would be most readily available for a general combination. And the micrometric results could be easily included, for they are essentially measures in rectangular coordinates, and they might be reduced in exactly the same way.

Cambridge Observatory :
1901 November 6.

The Determination of Selenographic Positions and the Measurement of Lunar Photographs.

[Second Paper.]

Determination of a first group of Standard Points by Measures made at the Telescope and on Photographs. By S. A. Saunder, M.A.

§ 1. *Recapitulation of First Paper.*

In a previous paper (*Monthly Notices*, vol. lx. p. 174) I called attention to the unsatisfactory state of our knowledge of the exact positions of the lunar formations, and to the increase in accuracy which might be obtained by measuring from the well-determined point *Mösting A* ; formulæ were developed for reducing the measures, and a few results were given.

It was also shown that by the measurement of such photographs as those now being taken at the Paris Observatory a great increase might be made in the number of points whose positions could be accurately determined without necessitating

an inordinate amount of computation. Formulæ were developed for reducing the measures on the assumption that the positions of a number of points were known with sufficient accuracy, but it was found that the accepted positions of "points of the first order" were not accurate enough to give the best results obtainable from the photographs, and a second method was developed depending partly on measures of the limb, partly on a combination of measures of the same points on two different photographs, somewhat analogous to the optical combination of two pictures by the stereoscope. Some tentative results were given, and in conclusion I expressed my intention of attacking other pairs of plates in the same way.

Further work on the same pair however showed that the constants obtained ought to be capable of improvement; there were signs of a progressive error, which I now know to have been due to an error of scale on one of the plates. This error was not of sufficient magnitude to vitiate the general conclusions drawn in the paper. I soon became convinced that no really satisfactory solution was likely to be obtained so long as any part of the work depended on measures of the limb. I then tried to determine the plate constants by an extension of what I have called the stereoscopic method, and when this failed, as will be described below, I had recourse to the telescope, and proceeded to determine the positions of a number of points surrounding *Mösting A* at distances not much exceeding 500'', this limit being imposed by the construction of my micrometer.

§ 2. *Telescopic Measures on the Moon.*

These measures were all made and reduced as described in § 3 to § 6 of the previous paper, the only change in detail being that, with the notation of § 4, when ξ' , η' , ζ' have been once measured the equation $z' = a_3 \xi' + b_3 \eta' + c_3 \zeta'$ has been used to compute the value of $1 - z' \sin s'$, the value of z' thus obtained being always sufficiently accurate for the purpose. x' , y' have then been determined from the equations at the bottom of p. 177.

$$x' = z \frac{1 - z' \sin s'}{1 - z \sin s'} - Q(1 - z' \sin s') \sin (P - C)$$

$$y' = y \frac{1 - z' \sin s'}{1 - z \sin s'} + Q(1 - z' \sin s') \cos (P - C)$$

This saves the steps required to compute the quantities denoted by x'_1 , y'_1 , z'_1 .

When the work was nearly completed I found that Dr. Franz of Breslau had already applied the method of measurement from *Mösting A* to determine the positions of eight other points, his results having been published in an Appendix to vol. xxxviii. of the Königsberg Observations a short time before my paper was read (see also *Monthly Notices*, vol. lxi. p. 265).

These measures have been of great use to me as will be explained below. The places expressed in terms of rectangular coordinates are :

Proclus	...	$\xi = +\cdot70222 \pm \cdot00012$	$\eta = +\cdot27698 \pm \cdot00020$
Macrobius <i>a</i>	...	$+\cdot61034 \pm 12$	$+\cdot33455 \pm 20$
Sharp <i>A</i>	...	$-\cdot45663 \pm 23$	$+\cdot73763 \pm 15$
Aristarchus	...	$-\cdot67551 \pm 17$	$+\cdot40201 \pm 21$
Gassendi <i>z</i>	...	$-\cdot65246 \pm 12$	$-\cdot28330 \pm 15$
Byrgius <i>A</i>	...	$-\cdot81612 \pm 17$	$-\cdot41561 \pm 20$
Nicolai <i>A</i>	...	$+\cdot29598 \pm 11$	$-\cdot67494 \pm 14$
Fabriceius <i>K</i>	...	$+\cdot46642 \pm 16$	$-\cdot72018 \pm 17$

The measures were all made with the Königsberg heliometer, and the observations extend over a period of six years. *Byrgius A* was measured on eight different nights ; the others depend each on from eleven to thirteen separate determinations.

An account of the measures of *Ptolemæus A* and *Triesnecker B*, kindly made for me by Professor Barnard, has already appeared (*Monthly Notices*, vol. lx. p. 540), together with the particulars of my own measures of these points (see also *Astronomical Journal*, vol. xxii. p. 33).

Four measures of *Dollond* are given in *Monthly Notices*, vol. ix. p. 182.

These, with *Müsting A* and those given below, complete the list of the positions used in the reduction of the photographic plates.

A complete set of measures consists, as described in § 6 of my previous paper, of six settings for position-angle, five double distances, and six more settings for position-angle. Occasionally a set has been interrupted by clouds or by failure of definition. If less than half the whole set had been made, no account has been taken of them ; if more than half, they have been reduced and allowed weight $\frac{1}{2}$ in the final means. No place depends on less than four complete sets or their equivalent.

The probable error of a coordinate derived from one complete set, as determined from the whole of the residuals, is $\cdot00022$, and hence the probable error of the mean of four such measures is $\cdot00011$, the geocentric value of which is $0''\cdot10$.

Results of micrometric measures made on the Moon with a 7-inch O.G.

Hesiodus B.

		Residual.		Residual.	
1899 Dec.	14	$\xi = -\cdot2670 + \cdot0001 \times \frac{1}{2}$	$\eta = -\cdot4553 + \cdot0002 \times \frac{1}{2}$	Wt. $\frac{1}{2}$	
	15	$-\cdot2673 - 2$	$-\cdot4554 + 1$		
1900 Feb.	11	$-\cdot2673 - 2$	$-\cdot4558 - 3$		
	Nov. 9	$-\cdot2664 + 7$	$-\cdot4554 + 1$		
1901 Apr.	1	$-\cdot2676 - 5$	$-\cdot4555 0$		
		$-\cdot2671$	$-\cdot4555$		

Nicollet.

		Residual.		Residual.
1900 Aug. 17	$\xi = -\cdot 1995 + \cdot 0003$		$\eta = -\cdot 3729 + \cdot 0001$	
Oct. 13	$-\cdot 1999 -$	1	$-\cdot 3732 -$	2
1901 Jan. 29	$-\cdot 1997 +$	1	$-\cdot 3732 -$	2
Feb. 1	$-\cdot 2001 -$	3	$-\cdot 3726 +$	4
	$-\cdot 1998$		$-\cdot 3730$	

Thibit A.

1900 Apr. 10	$\xi = -\cdot 0785 + \cdot 0007$		$\eta = -\cdot 3672 + \cdot 0002$	
13	$-\cdot 0795 -$	3	$-\cdot 3675 -$	1
May 7	$-\cdot 0792$	0	$-\cdot 3671 +$	3
Nov. 10	$-\cdot 0796 -$	4	$-\cdot 3676 -$	2
	$-\cdot 0792$		$-\cdot 3674$	

Abenezra b.

1900 Sept. 12	$\xi = +\cdot 1634 - \cdot 0001$		$\eta = -\cdot 3555 - \cdot 0003$	
Oct. 10	$+\cdot 1635$	0	$-\cdot 3546 +$	6
1901 Jan. 31	$+\cdot 1633 -$	2	$-\cdot 3557 -$	5
Feb. 28	$+\cdot 1636 +$	1	$-\cdot 3551 +$	1
	$+\cdot 1635$		$-\cdot 3552$	

Airy A.

1900 Sept. 12	$\xi = +\cdot 1276$	$\cdot 0000$	$\eta = -\cdot 2927 + \cdot 0002$	
Oct. 10	$+\cdot 1275 -$	1	$-\cdot 2929$	0
1901 Jan. 29	$+\cdot 1281 +$	5	$-\cdot 2927 +$	2
Mar. 3	$+\cdot 1274 -$	2	$-\cdot 2931 -$	2
	$+\cdot 1276$		$-\cdot 2929$	

Lubiniezky B.

1900 Apr. 13	$\xi = -\cdot 3863$	$\cdot 0000$	$\eta = -\cdot 2512 + \cdot 0003$	
Aug. 17	$-\cdot 3865 -$	2	$-\cdot 2515$	0
Oct. 14	$-\cdot 3865 -$	2	$-\cdot 2514 +$	1
1901 Jan. 2	$-\cdot 3861 +$	2	$-\cdot 2517 -$	2
	$-\cdot 3863$		$-\cdot 2515$	

Hipparchus C.

1900 Apr. 13	$\xi = +\cdot 1420 - \cdot 0003$		$\eta = -\cdot 1288 - \cdot 0003$	
May 7	$+\cdot 1424 +$	1	$-\cdot 1290 -$	5
Aug. 13	$+\cdot 1423$	0	$-\cdot 1283 +$	2
1901 Jan. 2	$+\cdot 1423$	0	$-\cdot 1281 +$	4
	$+\cdot 1423$		$-\cdot 1285$	

Euclides.

		Residual.		Residual.
1899	Dec. 13	$\xi = -\cdot4886$	$\cdot0000$	$\eta = -\cdot1279 + \cdot0004$
	15	$-\cdot4887 -$	1	$-\cdot1283$
1901	Feb. 11	$-\cdot4883 +$	3	$-\cdot1283$
	Nov. 10	$-\cdot4888 -$	2	$-\cdot1286 -$
		$-\cdot4886$		$-\cdot1283$

Hipparchus G.

1900	Nov. 9	$\xi = +\cdot1289 + \cdot0002$		$\eta = -\cdot0871 + \cdot0004$
	10	$+\cdot1290 +$	3	$-\cdot0871 +$
1901	Jan. 4	$+\cdot1285 -$	2	$-\cdot0880 -$
	Feb. 1	$+\cdot1284 -$	3	$-\cdot0878 -$
		$+\cdot1287$		$-\cdot0875$

Gambart A.

1900	Mar. 12	$\xi = -\cdot3205 + \cdot0005$		$\eta = +\cdot0173 + \cdot0003$
	13	$-\cdot3211 -$	1	$+\cdot0170 +$
	Apr. 10	$-\cdot3210$	0	$+\cdot0171 +$
	Nov. 9	$-\cdot3214 -$	4	$+\cdot0167 -$
		$-\cdot3210$		$+\cdot0170$

Cayley.

1900	Aug. 12	$\xi = +\cdot2599 - \cdot0004$		$\eta = +\cdot0691 + \cdot0001$
	13	$+\cdot2603$	0	$+\cdot0691 +$
1901	Jan. 30	$+\cdot2604 +$	1	$+\cdot0689 -$
	Feb. 28	$+\cdot2606 +$	3	$+\cdot0690$
		$+\cdot2603$		$+\cdot0690$

Hortensius.

1900	Aug. 17	$\xi = -\cdot4662$	$\cdot0000$	$\eta = +\cdot1127 - \cdot0001$
	Oct. 14	$-\cdot4662$	0	$+\cdot1128$
1901	Jan. 2	$-\cdot4657 +$	5	$+\cdot1134 +$
	30	$-\cdot4664 -$	2	$+\cdot1122 -$
	31	$-\cdot4666 -$	$4 \times \frac{1}{2}$	$+\cdot1127 -$
		$-\cdot4662$		$+\cdot1128$

Sosigenes a.

1900	Nov. 10	$\xi = +\cdot3141 - \cdot0004 \times \frac{1}{2}$		$\eta = +\cdot1357 + \cdot0008 \times \frac{1}{2}$
	Dec. 29	$+\cdot3149 +$	$4 \times \frac{1}{2}$	$+\cdot1353 +$
1901	Jan. 29	$+\cdot3147 +$	2	$+\cdot1348 -$
	Feb. 28	$+\cdot3145$	0	$+\cdot1346 -$
	Mar. 3	$+\cdot3143 -$	2	$+\cdot1346 -$
		$+\cdot3145$		$+\cdot1349$

Bode B.

			Residual.		Residual.
1900	Sept. 12	$\xi = -\cdot 0526 + \cdot 0003$		$\eta = +\cdot 1522 - \cdot 0003$	
	Oct. 13	$-\cdot 0528 +$	1	$+\cdot 1524 -$	1
1901	Jan. 31	$-\cdot 0530 -$	1	$+\cdot 1525$	0
	Feb. 1	$-\cdot 0534 -$	5	$+\cdot 1530 +$	5
		$-\cdot 0529$		$+\cdot 1525$	

Bode A.

1899	Dec. 15	$\xi = -\cdot 0203 - \cdot 0004$		$\eta = +\cdot 1570 + \cdot 0001$	
1900	Mar. 9	$-\cdot 0194 +$	5	$+\cdot 1569$	0
	12	$-\cdot 0198 +$	1	$+\cdot 1568 -$	1
	13	$-\cdot 0200 -$	1	$+\cdot 1569$	0
		$-\cdot 0199$		$+\cdot 1569$	

Bessel.

1900	Aug. 12	$\xi = +\cdot 2853 - \cdot 0003$		$\eta = +\cdot 3707 + \cdot 0003$	
	13	$+\cdot 2852 -$	4	$+\cdot 3709 +$	5
1901	Jan. 2	$+\cdot 2855 -$	1	$+\cdot 3697 -$	7
	4	$+\cdot 2864 +$	8	$+\cdot 3700 -$	4
	Apr. 1	$+\cdot 2856$	0	$+\cdot 3706 +$	2
		$+\cdot 2856$		$+\cdot 3704$	

§ 3. *Attempt to determine Absolute Places from a comparison of two Photographs only.*

The unsuccessful attempt to improve the constants for the plates denoted by the numbers II., III. in my previous paper, to which I referred in my first paragraph, was made as follows :—

The standard coordinates of a point ξ, η are connected with the plate coordinates x, y and the third coordinate $z = \sqrt{1 - x^2 - y^2}$, as shown in *Monthly Notices*, vol. lx. p. 187, by the equations

$$\xi = Ax + By + Cz$$

$$\eta = Dx + Ey + Fz$$

where A, B . . . are given as functions of the librations and of θ , the orientation of the réseau on the plate. All the coordinates are expressed as fractions of the Moon's radius, and referred to the centre of the Moon as origin.

If a small error $\delta\theta$ has been made in the assumed orientation it was shown (*Monthly Notices*, vol. lx. p. 189) that the corresponding errors in ξ, η are $(Bx - Ay) \delta\theta$ and $(Dx - Ey) \delta\theta$ respectively.

But errors may also have been made in $\alpha' \beta'$, the assumed réseau coordinates of the centre of the Moon's disc, and in r' , the assumed value of the radius, also in réseau units. Let $\delta\alpha'$, $\delta\beta'$, $\delta r'$ be the amounts of these errors in réseau units; $\delta\alpha$, $\delta\beta$, δr the same quantities expressed in terms of the Moon's radius.

Let x' , y' be the réseau coordinates of the point as actually read, *i.e.* referred to one corner of the réseau as origin.

$$\text{Then} \quad x = \frac{x' - \alpha'}{r'}, \quad y = \frac{y' - \beta'}{r'}$$

$$\begin{aligned} \therefore \delta x &= \frac{-r' \delta\alpha' - (x' - \alpha') \delta r'}{r'^2} \\ &= -\frac{\delta\alpha'}{r'} - \frac{x' - \alpha'}{r'} \frac{\delta r'}{r'} \\ &= -\delta\alpha - x \delta r \end{aligned}$$

$$\text{Similarly} \quad \delta y = -\delta\beta - y \delta r$$

$$\text{And since} \quad z = \sqrt{1 - x^2 - y^2}$$

$$\begin{aligned} \therefore \delta z &= -\frac{x \delta x + y \delta y}{z} \\ &= \frac{x}{z} \delta\alpha + \frac{y}{z} \delta\beta + \frac{1 - z^2}{z} \delta r \end{aligned}$$

And hence

$$\begin{aligned} \delta\xi &= A\delta x + B\delta y + C\delta z + (Bx - Ay)\delta\theta \\ &= \left(C \frac{x}{z} - A\right) \delta\alpha + \left(C \frac{y}{z} - B\right) \delta\beta + \frac{C}{z} \delta r - (Ax + By + Cz) \delta r \\ &\quad + (Bx - Ay) \delta\theta \\ &= \left(C \frac{x}{z} - A\right) \delta\alpha + \left(C \frac{y}{z} - B\right) \delta\beta + \left(\frac{C}{z} - \xi\right) \delta r + (Bx - Ay) \delta\theta \end{aligned}$$

Similarly

$$\delta\eta = \left(F \frac{x}{z} - D\right) \delta\alpha + \left(F \frac{y}{z} - E\right) \delta\beta + \left(\frac{F}{z} - \eta\right) \delta r + (Dx - Ey) \delta\theta$$

To determine these corrections thirty-seven points were measured on each of the two plates, the measures were reduced with the approximate values of α' , β' , r' , and θ already obtained, and then thirty-seven equations were made of the type

$$\xi_2 + \delta\xi_2 = \xi_3 + \delta\xi_3$$

and thirty-seven of the type

$$\eta_2 + \delta\eta_2 = \eta_3 + \delta\eta_3$$

where the suffixes indicate the plate on which the measures were made.

These gave seventy-four equations for determining the eight quantities, $\delta a_2, \delta \beta_2, \delta r_2, \delta \theta_2, \delta a_3, \delta \beta_3, \delta r_3, \delta \theta_3$.

When these were combined by least squares, and the normal equations solved by Gauss' method of elimination, it was found that so long as one of each of the quantities $\delta a, \delta \beta, \delta \theta, \delta r$ remained in the equations the coefficients of the first term in each equation ranged between 7.8 and 36.9, but that so soon as all the quantities belonging to one plate had been eliminated the first coefficient of the next equation fell to between .01 and .04, according to the order of elimination, showing that the values which would be given by the solution would be of very low weight, whilst on pushing to a numerical result a position of the centre was obtained which was known to be in error by something like three seconds of arc.

The reason for this may be seen by considering any pair of the normal equations; if, for example, we take those in $\delta a_2, \delta a_3$, these, as written in the usual form, were

$$\begin{aligned} 37.0643\delta a_2 - 36.1251\delta a_3 + .1831\delta \beta_2 + 9.3175\delta \beta_3 + 1.5436\delta \theta_2 \\ + 1.8799\delta \theta_3 - 2.7882\delta r_2 - 5.1871\delta r_3 + .0040061 = 0 \\ - 36.1251\delta a_2 + 37.7490\delta a_3 - 9.8515\delta \beta_2 + .2599\delta \beta_3 - 1.1806\delta \theta_2 \\ - .5903\delta \theta_3 + 2.9563\delta r_2 + 6.0044\delta r_3 - .0037328 = 0 \end{aligned}$$

If we put $\delta a_2 - \delta a_3 = 2A$, $\delta a_2 - \delta a_3 = 2a$, the first two terms become

$$\begin{aligned} .9392A + 73.1894a \text{ in the first equation,} \\ 1.6239A - 73.8741a \text{ in the second equation,} \end{aligned}$$

whence we see that while the differences are well determined the sums have very small coefficients; and the same holds true for all the pairs of normal equations.

The interpretation is that the unavoidable errors of observation were such that the difference between the librations was not sufficient to give more than a rough approximation to the absolute values of the required constants.

§ 4. *Formation of Equations by Combination of the Telescope and Photographic Measures.*

The work, however, was not entirely lost, for by stopping the elimination when only half effected it was possible (as explained in *Chauvenet*, vol. ii., Appendix, § 52) to obtain four good equations:

$$\begin{aligned} \delta a_2 &= P\delta a_3 + Q\delta \beta_3 + R\delta r_3 + S\delta \theta_3 + T \pm .000031 \\ \delta \beta_2 &= P'\delta a_3 + Q'\delta \beta_3 + R'\delta r_3 + S'\delta \theta_3 + T' \pm .000031 \\ \delta r_2 &= P''\delta a_3 + Q''\delta \beta_3 + R''\delta r_3 + S''\delta \theta_3 + T'' \pm .000067 \\ \delta \theta_2 &= P'''\delta a_3 + Q'''\delta \beta_3 + R'''\delta r_3 + S'''\delta \theta_3 + T''' \pm .000067 \end{aligned}$$

where P, Q, R . . . were obtained numerically, and the probable errors are expressed in terms of the Moon's radius, the geocentric value of the largest being 0".06.

These equations were used to transform the linear equations

$$\delta\xi_2 = F(\delta\alpha_2, \delta\beta_2, \delta\theta_2, \delta r_2) \quad \delta\eta_2 = f(\delta\alpha_2, \delta\beta_2, \delta\theta_2, \delta r_2)$$

found in § 3 into equivalent linear equations

$$\delta\xi_2 = F_1(\delta\alpha_3, \delta\beta_3, \delta\theta_3, \delta r_3) \quad \delta\eta_2 = f_1(\delta\alpha_3, \delta\beta_3, \delta\theta_3, \delta r_3)$$

in which the coefficients are functions of the position of the point ξ, η , as well as of the constants P, Q, R . . .

With these equations measures made upon Plate II. could be used to determine the corrections to the constants of Plate III., and as Plate II. was taken a little after first quarter, Plate III. a little before third quarter, the two together show the whole disc, with a slight overlap near the central meridian.

But in order to find these corrections it was necessary to know the positions of a number of points on the Moon's surface, and it had already been shown (*Monthly Notices*, vol. lx. p. 184) that those previously found as "points of the first order" were not good enough for the purpose. It was at this stage that use was made of the places found at the telescope.

The available material consisted of the positions of *Mösting A*, from which all the others had been measured, 19 points within about 500" of *Mösting A* measured by myself, two of which had been also measured by Professor Barnard, and the 8 points, all near the limb, measured by Dr. Franz. Two of these 8, *Aristarchus* and *Byrgius A*, were, in consequence of the brightness of the surrounding regions, too poorly defined to be measurable on the photographs. I had therefore a total of 26 points, of which 8 appeared on Plate II. only, 8 on Plate III. only, and 10 were shown on both plates. These were all the centres of regular circular formations. I had also measured the cones of *Herschel* and *Manilius*, but in consequence of the difficulty in locating the summit of a mountain I decided not to employ any points of this nature.

The selected points having been carefully measured on the photographs, the corresponding values of ξ, η were computed for each, as well as the coefficients of the different terms in the expressions for $\delta\xi, \delta\eta$, the approximate values of the plate constants being used for the purpose.

Equating the values of $\xi + \delta\xi, \eta + \delta\eta$ found from the photographs to the corresponding values of ξ, η found at the telescope, there were 72 equations for the four quantities $\delta\alpha_3, \delta\beta_3, \delta r_3, \delta\theta_3$. But these were not all dealt with in one group.

§ 5. *Comparison of Constants obtained from Dr. Franz's Measures with those obtained from my own.*

The first point about which I was anxious to satisfy myself was as to the accordance between the positions which I had found and those measured by Dr. Franz. I had already found (*Monthly Notices*, vol. lx. p. 540) a satisfactory agreement between my own measures and measures of the same points by Professor Barnard, but in this case the distances measured had been under 160". My own micrometer screw had begun to show signs of wear, evidenced by a want of constancy of the error of runs in different parts of the scale, and I felt anxious as to any possible effect of this upon the measures.

I therefore picked out the 60 conditional equations derived from the positions of *Mösting A* and of the points which I had measured myself, and from these determined the corrections to the plate constants.

I then dealt in the same way with the 16 equations obtained from the positions of *Mösting A* and of the points measured by Dr. Franz.

The following table shows the results of the comparison. The values of δa_3 , $\delta \beta_3$, δr_3 are expressed first in terms of the Moon's radius as unit, and secondly in seconds of arc at mean distance of the Moon; $\delta \theta_3$ is expressed first in circular measure, and secondly in seconds of arc.

	δa_3	$\delta \beta_3$	δr_3	$\delta \theta_3$
Difference between absolute values of corrections found from the two groups of equations	... '00023 = 0".21	'00022 = 0".21	'00030 = 0".28	'00002 = ".4
P.E. from measures by S. A. S	... '00005 = 0".05	'00005 = 0".05	'00015 = 0".14	'00015 = 31
P.E. from measures by J. F.	... '00011 = 0".10	'00010 = 0".09	'00015 = 0".14	'00014 = 29

The first point to which attention was directed was that the differences between the values of δa_3 , $\delta \beta_3$ were greater than the sum of the probable errors, whilst the probable errors from my own points were about half those from Dr. Franz's points. The fact that my values depended on nearly four times as many conditional equations would explain the difference in probable error, but the difference in the values found I attributed chiefly to the fact that the points which I had measured at the telescope were more clearly defined on the photographs than those which Dr. Franz had measured, so that the photographic measures of my points were more trustworthy than those of Dr. Franz's. It did not appear likely that the position of the centre as determined from my points alone would be subject to any systematic error, and, moreover, when the whole of the 72 conditional equations were solved in one group the probable errors of δa_3 , $\delta \beta_3$ were '000059 and '000058 respectively, a little greater than those determined from my points alone. It therefore appeared

that the centre would be best determined by considering only the equations of the first group.

The want of definition of Dr. Franz's points upon the photographs would also affect the values of the radius and orientation derived from them; but this was compensated by their greater distance from the centre, and the probable errors from the two groups were practically the same. But in the normal equations for δr_3 the coefficient of δa_3 was nearly as great as that of δr_3 . This was found to be due, as to *Mösting A* and the points which I had measured, to a relation between the libration and the position of the terminator in consequence of which the coefficients of δa_3 , δr_3 retained the same sign in all the equations of the ξ group given by Plate II. As to the points measured by Dr. Franz the fact that four of them were on one side of the central meridian and only two of them on the other operated in the same way. The result was that the value found for δr_3 depended largely upon that assigned to δa_3 , whilst the normal equation for δa_3 was such that the value of δa_3 was only slightly affected by that of δr_3 . When this was noticed the normal equations in δr_3 , $\delta \theta_3$ given by my points and those given by Dr. Franz's points were solved with the same values of δa_3 , $\delta \beta_3$. The difference between the two values found for δr_3 was then reduced to $\cdot 00015$ of the Moon's radius = $0''.14$. The difference in the scales derived from the two sets of measures amounted therefore to but little more than one part in 7,000, with which agreement I was quite satisfied, and from which it would appear that the wear of the screw had not been sufficient to introduce sensible error into the results.

The two values of $\delta \theta_3$ were practically coincident, though the probable errors show that this was partly fortuitous.

§ 6. *Final Solution of the Equations.*

Having decided that the whole 72 equations should be employed in the determination of δr_3 , $\delta \theta_3$, whilst the values of δa_3 , $\delta \beta_3$ should depend upon the group of 60 only, the proper course would have been to express δr_3 , $\delta \theta_3$ as functions of δa_3 , $\delta \beta_3$ from the 72 equations, to substitute these values in the group of 60, and to solve the resulting equations for δa_3 , $\delta \beta_3$. This, however, would have involved a considerable addition to the labour of computation, which was already very great, whilst the effect upon the solution would have been almost imperceptible. The greatest difference between the solutions obtained entirely from one set or entirely from the other set of equations amounted only to $0''.2$. I therefore compromised matters by taking the two normal equations in δa_3 , $\delta \beta_3$ from the group of 60 conditional equations, the two in δr_3 , $\delta \theta_3$ from the whole group of 72, and accepted as final the values resulting from their simultaneous solution.

Having determined these the corrections δa_2 , $\delta \beta_2$, δr_2 , $\delta \theta_2$ were given by the equations given at the beginning of § 4, and a set of constants was determined which would enable the standard coordinates to be found for any point measurable on either plate.

It should be noted that the coordinates of the 10 points measured at the telescope and shown on both plates enter each into two conditional equations, being equated separately to the corresponding coordinate from each photograph; but those of the 16 points shown on one plate only enter each into only one conditional equation, and so have less weight in the normal equations than the others. But I believe that the principal errors at present are in the photographic measures, and that consequently the course adopted is justifiable.

The advantages which would seem to be secured by expressing the constants of one plate in terms of those of the other are, first that a greater number of conditional equations is obtained than could be secured from either plate alone, and secondly that the use of two plates taken under different librations and oppo-

Comparison between Places derived from Paris Plates II. and III.,
Professor Barnard and

	Values of ξ .					
	Plate II.	Plate III.	II.-III.	Mean of II. & III.	Tele- scopic.	Photo- graphic -Tele- scopic.
Lilius (sm. crater)*	+ '0617	+ '0620	- '0003	+ '0618	...	...
Licetus (sm. crater)†	+ '0231	+ '0221	+ 10	+ '0226	...	...
Fabricius K	+ '4666	...	...	...	+ '4664	+ '0002
Licetus H	+ '0383	+ '0381	+ 2	+ '0382	...	...
Stöfler f	+ '0641	+ '0633	+ 8	+ '0637	...	...
Nicolai A	+ '2966	...	...	...	+ '2960	+ 6
Stöfler K	+ '0567	+ '0566	+ 1	+ '0567	...	...
Orontius d	- '0829	- '0829	0	- '0829	...	...
Walter a	+ '0093	+ '0090	+ 3	+ '0091	...	...
„ (sm. crater)‡	- '0025	- '0028	+ 3	- '0026	...	...
Hesiodus B	...	- '2680	...	...	- '2671	- 9
Purbach A	- '0295	- '0296	+ 1	- '0295	...	...
Nicollet	...	- '2003	...	...	- '1998	- 5
Thebit A	- '0789	- '0789	0	- '0789	- '0792	+ 3
Abenezra b	+ '1638	+ '1639	- 1	+ '1638	+ '1635	+ 3
Arzachael A	- '0247	- '0247	0	- '0247	...	...
Airy (sm. crater)§	+ '0743	+ '0746	- 3	+ '0745	...	...
„ A	+ '1277	+ '1279	- 2	+ '1278	+ '1276	+ 2

* The most westerly of three confluent craters of which the most easterly is b.

† A small round crater west of f and d.

‡ The isolated crater on the south-east part of the floor.

§ To the east of Airy, almost in contact with the wall.

site illuminations tends to eliminate any systematic error depending upon either of these conditions. It is perhaps open to question whether the same amount of labour spent upon measuring and reducing several different plates independently, and taking the means of the resulting places, would not have given a closer approximation to the true values of the coordinates; but the greater part of the work was performed in the endeavour to obtain absolute places from a comparison of two photographs only, without appealing to telescopic measures at all. It was only when this failed that the method finally adopted suggested itself, and I think that the labour spent upon the reductions from that period has yielded an adequate return.

In the following table I give a comparison of the places finally obtained from the two plates with each other, and with those obtained at the telescope. The suffixes as before refer to the Plates II., III. respectively. The column headed "Authority" refers to the telescopic measures, and the initial B. denotes Professor Barnard, F. Dr. Franz, and S. myself.

Heliometer Measures by Dr. Franz, and Filar Micrometer Measures by S. A. Saunder.

Values of η .						
Plate II.	Plate III.	II. - III.	Mean of II. & III.	Telescopic.	Photographic - Telescopic.	Authority.
-.7960	-.7964	+ .0004	-.7962	...	...	
-.7285	-.7287	+ 2	-.7286	...	...	
-.7197	...	...	...	-.7202	+ .0005	F.
-.7182	-.7182	0	-.7182	...	...	...
-.6774	-.6773	- 1	-.6773	...	...	...
-.6742	...	...	...	-.6749	+ 7	F.
-.6352	-.6352	0	-.6352	...	...	...
-.6345	-.6345	0	-.6345	...	...	...
-.5935	-.5935	0	-.5935	..	...	...
-.5546	-.5545	- 1	-.5545	...	...	...
...	-.4556	...	...	-.4555	- 1	S.
-.4405	-.4406	+ 1	-.4405	...	...	...
...	-.3725	...	...	-.3730	+ 5	S.
-.3677	-.3673	- 4	-.3675	-.3674	- 1	S.
-.3550	-.3548	- 2	-.3549	-.3552	+ 3	S.
-.3903	-.3903	0	-.3903	...	...	...
-.3025	-.3024	- 1	-.3025	...	...	...
-.2926	-.2926	0	-.2926	-.2929	+ 3	S.

	Values of ξ .					Tele- scopic.	Photo- graphic —Tele- scopic.
	Plate II.	Plate III.	II.—III.	Mean of II. & III.			
Gassendi ε	...	—'6522	...	...	—'6525	+	3
Alpetragius B	—'1152	—'1156	+ 4	—'1154	...	...	
Lubiniezky B	...	—'3864	...	...	—'3863	—	I
Dollond	+ '2456	...	...	...	+ '2458	—	2
Ptolemæus A	—'0139	—'0140	+ 1	—'0140	—'0142	+	2
"	"	"	"	"	—'0139	—	I
Halley	+ '0999	+ '0994	+ 5	+ '0996	...	...	
Hipparchus C	+ '1422	+ '1426	— 4	+ '1424	+ '1423	+	I
Euclides	...	—'4892	...	...	—'4886	—	6
Hipparchus L	+ '1560	+ '1565	— 5	+ '1563	...	...	
" G	+ '1288	+ '1292	— 4	+ '1290	+ '1287	+	3
" n	+ '0873	+ '0871	+ 2	+ '0872	...	...	
Mösting A	—'0904	—'0904	0	—'0904	—'0900	—	4
Hipparchus H	+ '0526	+ '0528	— 2	+ '0527	...	...	
Gambart A	...	—'3209	...	...	—'3210	+	I
Triesnecker B	+ '0068	+ '0067	+ 1	+ '0067	+ '0072	—	5
"	"	"	"	"	+ '0073	—	6
Cayley	+ '2602	...	...	...	+ '2603	—	I
Murchison A	+ '0199	+ '0199	0	+ '0199	...	...	
Hortensius	...	—'4660	...	...	—'4662	+	2
Sosigenes α	+ '3144	...	...	...	+ '3145	—	I
Bode B	—'0531	—'0528	— 3	—'0529	—'0529		0
" A	—'0201	—'0200	— 1	—'0201	—'0199	—	2
Proclus	+ '7022	...	...	...	+ '7022		0
Macrobius α	+ '6107	...	...	...	+ '6103	+	4
Conon	+ '0324	+ '0326	— 2	+ '0325	...	...	
Bessel	+ '2865	...	...	...	+ '2856	+	9
Aratus	+ '0727	+ '0728	— 1	+ '0728	...	...	
Archimedes A	—'0983	—'0983	0	—'0983	...	...	
" c	—'0221	—'0223	+ 2	—'0222	...	...	
" b	—'0268	—'0276	+ 8	—'0272	...	...	
Kirch	—'0754	—'0755	+ 1	—'0754	...	...	
Cassini b	+ '0521	+ '0520	+ 1	+ '0521	...	...	
" A	+ '0635	+ '0635	0	+ '0635	...	...	
" (small crater)*	+ '0495	+ '0492	+ 3	+ '0494	...	...	
Piazzi Smyth	—'0416	—'0418	+ 2	—'0417	...	...	
Sharp A	...	—'4560	...	...	—'4566	+	6

* Just outside Cassini to the north-east.

Values of η .

Plate II.	Plate III.	II.-III.	Mean of II. & III.	Tele- scopic.	Photo- graphic - Tele- scopic.	Autho- rity.
...	- '2829	...	...	- '2833	+	4 F.
- '2604	- '2605	+ I	- '2605	...	...	...
...	- '2516	...	...	- '2515	+	I S.
- '1817	...	...	...	- '1817	0	S.
- '1480	- '1480	0	- '1480	- '1479	-	I B.
"	"	"	"	- '1478	-	2 S.
- '1390	- '1394	+ 4	- '1392	...	...	...
- '1285	- '1285	0°	- '1285	- '1285	0	S.
...	- '1279	...	...	- '1283	+	4 S.
- '1190	- '1192	+ 2	- '1191	...	...	...
- '0872	- '0872	0	- '0872	- '0875	+	3 S.
- '0843	- '0841	- 2	- '0842	...	...	...
- '0558	- '0556	- 2	- '0557	- '0556	-	I F.
- '0386	- '0382	- 4	- '0384	...	...	...
...	+ '0167	...	...	+ '0170	-	3 S.
+ '0205	+ '0204	+ I	+ '0204	+ '0205	-	I B.
"	"	"	"	+ '0206	-	2 S.
+ '0690	...	...	...	+ '0690	0	S.
+ '0697	+ '0697	0	+ '0697	...	...	...
...	+ '1130	...	...	+ '1128	+	2 S.
+ '1353	...	...	...	+ '1349	+	4 S.
+ '1523	+ '1520	+ 3	+ '1521	+ '1525	-	4 S.
+ '1564	+ '1568	- 4	+ '1566	+ '1569	-	3 S.
+ '2772	...	...	...	+ '2770	+	2 F.
+ '3342	...	...	...	+ '3345	-	3 F.
+ '3697	+ '3693	+ 4	+ '3695	...	...	...
+ '3701	...	...	...	+ '3704	-	3 S.
+ '4004	+ '4003	+ I	+ '4003	...	...	...
+ '4699	+ '4699	0	+ '4699	...	...	...
+ '5238	+ '5237	+ I	+ '5238	...	...	...
+ '5702	+ '5697	- 5	+ '5699	...	...	...
+ '6321	+ '6321	0	+ '6321	...	...	...
+ '6422	+ '6422	0	+ '6422	...	...	...
+ '6488	+ '6489	- I	+ '6488	...	...	...
+ '6601	+ '6603	- 2	+ '6602	...	...	...
+ '6671	+ '6668	+ 3	+ '6670	...	...	...
...	+ '7380	...	...	+ '7376	+	F.

§ 7. *Discussion of Results obtained.*

It is satisfactory to note that no two positions of the same point are separated by as much as a second of arc, though in two or three cases this limit is approached. Some part of the discordances found may be real, depending, as has been shown by Dr. Franz, upon the elevation above or depression below the surface of the sphere most nearly representing the figure of the Moon. But it must be borne in mind that concordance between the places derived from the two photographs does not prove the accuracy of those places, otherwise the attempt to determine absolute places by securing such concordance would have succeeded. The real test is to be found in the agreement of places derived from the photographs with those obtained at the telescope.

The mean distance between these positions for the six points measured by Dr. Franz is $0''.53$, these points being shown on only one plate each. For the nineteen points measured by myself there are twenty-eight photographic measures on the two plates, and the mean of the twenty-eight divergences is $0''.38$, whilst for the ten points which have been measured on both plates the mean divergence between the telescopic and the mean photographic places is $0''.32$. The mean divergence for the eighteen points measured on Plate II. is $0''.37$; for the eighteen points measured on Plate III. it is $0''.42$.

The greater divergence exhibited by the six points measured by Dr. Franz is, I think, chiefly due to the fact that they are not well defined on the photographs. There is not one of them that I should have selected as a really satisfactory point for accurate measurement on these particular plates, and I am pleased to find that the divergence is no greater. This I think may be taken as a satisfactory indication that the scale and orientation are well determined, so that divergences do not sensibly increase as we recede from the centre. I think also it may be fairly concluded from the whole comparison that the position of a point on any part of the surface may be determined from a single photograph on which it is well defined with an error less than one second of arc.

In estimating the advance thus made towards the accurate determination of selenographic positions it must be remembered that we are not dealing with stellar points; that in the most favourable cases, such as those here considered, we have to estimate the centre of a formation seldom less than $5''$ or $6''$ in diameter, and frequently more, subject to all the confusion produced by a transition from dazzling brightness to the blackest shade, and that therefore it is not to be expected that the position of a lunar formation should be capable of as accurate determination as that of a fixed star. I may perhaps be pardoned if I again call attention to the fact that the mean

distance between two positions of the same point found by Mädler and Lohrmann respectively, as determined from the eleven points measured as "points of the first order" not less than five times by each, was $5''\cdot2$, or almost exactly ten times the average discordance between the six points measured by Dr. Franz and my own determination of their positions, each from one photograph only, under conditions which were certainly not too favourable to the photographs.

The accuracy now attained is, I think, sufficient for such purposes as the preparation of large-scale maps, the computation of the Sun's altitude as required for determining the heights of mountains, and for the great problem to which these are subsidiary, the detection of possible changes on the surface.

But of greater importance than the increase of accuracy in the determination of a few isolated points is the enormous increase in the number of points which can now be accurately measured. With the old methods the computations required for the determination of "points of the first order" were so tedious that rougher methods were devised for determining points of the "second" and "third order," as they were called. These carried with them all the errors of the points of the first order from which they were measured, whilst introducing others due to the methods adopted for the reduction.

Beer and Mädler's map was based on 105 points of the first order, Schmidt's on 157. As a foundation for the outline maps which are being issued by the British Astronomical Association, I have already measured seventy-four points on an area equal to $\cdot014$ of the projection of the visible hemisphere (*B.A.A. Memoirs*, vol. x., 5th Report of the Lunar Section). This area is the corner of the S.W. quadrant near the centre of the disc, and includes the whole of *Hipparchus* and portions of the neighbouring formations. It was selected as a suitable region for the commencement of a scheme for re-mapping the whole surface, and if this is well taken up it is proposed to gradually extend the area. More points might have been measured on either of the plates, but I preferred to limit myself to those which could be measured with a fair approach to accuracy on both, and I have every reason to believe that these seventy-four points are determined with errors very little in excess of those in the list now given. With more photographs it will be possible to measure other points in the same area, as well as to increase the accuracy of those already measured. Some regions will perhaps give an even closer aggregation of measurable points, whilst the large seas will probably not give so many. But the density of distribution on the area already measured corresponds to more than 5,000 on the whole hemisphere. These will again serve as points of reference by which to fix the still more numerous points not at present shown in the photographs, but easily seen in a small telescope, and in which we are most likely to detect evidences of those changes which have been

so strongly suspected by the most trustworthy selenographers, and so ruthlessly rejected by the majority of the writers of astronomical text-books.

§ 8. *Focal Length of the Paris Telescope and Scale of the Photographs.*

In § 13 of my previous paper I gave the values found for the focal length of the photographic object glass of the Paris great Equatorial Coudé from the measures then made. These were vitiated by an erroneous assumption as to the size of the réseau square employed, which was really 5.040 mm., instead of 5.000 mm., as well as by the errors attendant upon limb measures. With the scale values now adopted it appears that a side of the réseau square on Plate II. corresponds to 57''·55, and on Plate III. to 57''·59. The values which I obtain for the focal length of the telescope are 18.064 m. on 1899 February 18, and 18.052 m. on 1895 September 9. The temperatures on these two nights were probably very different, but the difference of focal length is of the opposite sign to that which would be expected from this cause alone.

§ 9. *Conclusion.*

The computations have been almost entirely performed with a "Brunsviga," which is admirably adapted, not only for computing expressions of the form $Ax + By + Cz$, of which a large part of the work has consisted, but also for forming the normal equations in a least square solution, and for the eliminations according to Gauss' method. The computations have been carried to not less than five places throughout; where there appeared to be special danger of cumulative error more figures have been retained. The values of ξ , η given by the photographic plates have been doubly computed—first by applying the differential corrections to the values computed with the approximate plate constants, and secondly directly from the measured coordinates with the corrected values of the plate constants.

I should hope that the positions now obtained are sufficient both in number and in accuracy to allow of the determination of the constants of a plate by the method proposed by Professor Turner, and described in § 9 of my previous paper, by which a considerable reduction is effected in the amount of computation required. My next step will be to endeavour to reduce a plate in this way. If I am successful progress in the future will be more rapid than it has been in the past.

The plates, as stated in my previous paper, are positive copies on glass, kindly made for me by Mr. Bellamy, of the University Observatory, Oxford, from Paris negatives, for which I am indebted to the generosity of M. Loewy.

The photographic measures were made with an astrographic micrometer of the pattern in use at Greenwich and Oxford, the property of the University Observatory, Oxford, and kindly lent me by Professor Turner, who has not only placed the resources of the Observatory at my disposal, but has supplemented them with a large share of personal advice and assistance, for all of which my most hearty thanks are due.

§ 10. *Comparison of Results with those in Breslau*
"Mitteilungen," Vol. I.

Since this paper was written copies have been received in England of the first volume of the *Mitteilungen* of the Breslau Observatory, in which Dr. Franz gives the places of 150 lunar formations as determined by the measurement of five Lick photographs, and it has been with great interest that I have compared his methods and results with my own. In what may be considered the essentials of the methods we agree: we have both made our work independent of direct measures of the limb, and have determined the constants of our plates by comparing the photographic measures of a number of points with the positions of the same points determined by telescopic measures from *Mösting A*; but in the details there are many points of difference.

Professor Franz's telescopic measures have been made with the heliometer, my own with the filar micrometer, the bisection of two formations with the latter instrument being an entirely different operation from the superposition of their images with the former. Dr. Franz has been able to measure eight points near the limb. I have been obliged to measure a greater number nearer the centre, although I have been able to avail myself to a limited extent of his published results, as explained above.

The photographs which Dr. Franz has measured have all been taken near the time of full Moon, so that a great number of the objects measured have appeared only as light spots, no detail being visible. My photographs show but little more than half the disc illuminated, and I have not measured a crater unless a considerable portion of the wall could be made out.

In measuring the photographs Dr. Franz has bisected the craters with a thread moved by a screw, the reading of which, added to that of an accurate scale on which the microscope was carried, gave the measured coordinate. I have measured with a réseau and a graduated cross in the eyepiece, the setting being made either by getting equal readings on opposite arms where they cut the crater walls, or by setting the crater symmetrically in one of the squares formed by the longer (fifth or tenth) division lines.

The reductions have also been performed by quite independent methods.

It would thus seem unlikely that the two sets of places should be affected by the *same* systematic errors.

In order to compare the results it was necessary for me to give relative weights to my own photographic and telescopic measures. I find that the probable error of a coordinate measured on one photographic plate is $0''.09$, as determined from the divergences between the two plates. The corresponding probable error for the mean of four telescopic measures was found in § 2 to be $0''.10$. But it has already been pointed out that the measure of the discordance between the two plates may give too high an estimate of the value of the absolute results obtained, since errors in the constants give systematic errors in the results, and the method of solution renders it probable that these will be in the same direction in the two plates. I have therefore given to a coordinate obtained from one plate half the weight of the mean of four telescopic measures. I may note in passing that Dr. Franz gives no weight to his photographic measures as compared with those made with the heliometer.

In addition to *Mösting A* and the six points already referred to, there are seventeen points that Dr. Franz and I have both measured. In the following table I compare the positions which result from this combination of my own measures of these points with those found by Dr. Franz :—

	J. F.	Values of ξ . S. A. S.	F.-S.	J. F.	Values of η . S. A. S.	F.-S.
Nicollet	-1997	-2000	+0003	-3732	-3728	-0004
Thebit A	-0794	-0791	- 3	-3677	-3674	- 3
Airy (sm. crater)	+0746	+0745	+ 1	-3028	-3025	- 3
Airy A	+1278	+1277	+ 1	-2930	-2928	- 2
Lubiniezky B	-3859	-3863	+ 4	-2520	-2515	- 5
Ptolemæus A	-0138	-0140	+ 2	-1480	-1479	- 1
Hipparchus C	+1422	+1423	- 1	-1288	-1285	- 3
Euclides	-4880	-4888	+ 8	-1287	-1282	- 5
Hipparchus G	+1289	+1288	+ 1	-0874	-0874	0
Gambart A	-3211	-3210	- 1	+0167	+0169	- 2
Cayley	+2599	+2603	- 4	+0688	+0690	- 2
Sosigenes α	+3139	+3145	- 6	+1351	+1350	+ 1
Bode B	-0530	-0529	- 1	+1520	+1523	- 3
„ A	-0198	-0200	+ 2	+1562	+1568	- 6
Bessel	+2855	+2859	- 4	+3699	+3703	- 4
Aratus	+0724	+0728	- 4	+4005	+4003	+ 2
Archimedes A	-0984	-0983	- 1	+4698	+4699	- 1

In no case does the discordance between two positions amount to $1''$ of arc (geocentric). The only formation for which this value is approached is *Euclides*, which was not well defined on

Dr. Franz's photographs. This one may conjecture to have been due to the brightness of its surroundings.

The comparison reveals a systematic difference of $\cdot0002$ in the ordinates, but is consistent with the supposition that the differences in the abscissæ are accidental.

This difference in the ordinates is slightly increased when the comparison is made between Dr. Franz's positions and my telescopic positions only, whilst it would have practically disappeared from the photographic measures had I adopted the ordinate of the centre derived in § 5 from the photographic measures of the six points measured with the heliometer by Dr. Franz.

The result therefore justifies my rejection of these photographic measures so far as the abscissa of the centre is concerned, but gives reason for suspecting a real systematic difference of about $0''.2$ in the ordinates between Dr. Franz's positions found with the heliometer and mine found with the filar micrometer.

The mean geocentric distance between the positions of the same point, as found by Dr. Franz and myself, is $0''.39$, or less than one-thirteenth of the corresponding mean distance between positions found by Lohrmann and Mädler. There was also a systematic difference of a different kind between the ordinates found by these observers about fifteen times as great as that between Dr. Franz and myself, Lohrmann placing his points further from the equator than Mädler.

That small systematic differences should appear is not surprising. Dr. Franz calls attention to their existence between different parts of his own work, and I have already noted the same feature with regard to my work in the *Fifth Report of the Lunar Section of the British Astronomical Association*.

It says much for the general accuracy of the method that both these and the accidental differences should be as small as they are; and it is a great gratification to me to find my own work in such close agreement with that of one who has, I believe, given more attention to selenographic measurements than any other living astronomer, and to whom we owe the best determinations both of the physical libration and of the figure of the Moon.

Recent Observations of the Position of Nova Aurigæ with the 40-inch Telescope of the Yerkes Observatory. By E. E. Barnard.

Shortly after the announcement of Dr. Anderson's discovery of *Nova Aurigæ* in 1892 February Mr. Burnham made a careful set of measures of its position with reference to some thirteen stars near it.

The results of this work are given by him in a paper in

Monthly Notices for 1892 April. In the same publication for 1892 December he gives his observations of the brightness of the *Nova* as it slowly faded out. These observations were made with the 36-inch equatorial of the Lick Observatory. From March 13 of that year, when his determinations of magnitude began, until April 26, when proximity to the Sun prevented further work, the star had diminished from the 8th to the 16th magnitude. In Mr. Burnham's opinion it could not have been seen with the 36-inch for more than one week longer even had the star been favourably placed for observation. After March 13 the decrease in light was essentially uniform at the rate of 0.18 magnitude a day.

In August of the same year the star had brightened up again to between the 10th and 11th magnitudes, and had undergone a remarkable physical change, as shown by both visual and spectroscopic observations, indicating that the *Nova* then possessed the characteristics of a planetary nebula.

This star has faded considerably in the past few years, and is now of about the 13th magnitude.

In *Astronomische Nachrichten*, 3143, 3238, and 3279, I have given micrometer measures of the *Nova* with respect to two of Mr. Burnham's comparison stars, E and F of his diagram in *Monthly Notices* for 1892 April. These measures showed that in the two years during which the star was under observation no appreciable change occurred in its position, there being no evidence of parallax or proper motion, implying an enormous distance from the Earth.

My own measures were also in close agreement with those of Mr. Burnham.

The *Nova* has been examined a few times with the 40-inch, and I have in the past winter secured some further measures of its position with the large telescope. These show that no change has occurred in its place in the past eight years.

Mr. Newall has questioned the nebulous condition of the star, though the spectroscope has shown that it has the spectrum of a planetary nebula.

At the second appearance in 1892, when it had brightened up from the 16th, in May, to the 10th or 11th magnitude in August, it was found to have undergone a change both in appearance and in its spectrum. It had become bluish-white in colour, though in the earlier stages of its first appearance it was yellowish, and had seemed to grow redder as it diminished in brightness. It had also the appearance of a nebulous star.

In the present observations with the 40-inch the star was much fainter than at the measures with the 36-inch. On 1900 November 3 a direct estimation made it of the $13\frac{1}{2}$ magnitude, while from a comparison with the stars near it, whose magnitudes had been determined by Mr. Burnham, I found it to be of the 13.0 magnitude on Burnham's scale of magnitudes. On this date a determination of the difference of focus between the *Nova*

and the star F made the focus of the former 0·3 inch further out than for F. It was not as well defined as the stars.

During these observations the position of the *Nova* was again measured with reference to E and F on some five nights. The results are given as follows:—

Positions of Nova Aurigæ in 1900.

A and F.

Nov.	3	1900·842	32°46	84''96
	5	·847	32°52	...
	22	·894	32°73	84·87
	26	·905	32°64	84·93
Dec.	8	·937	32°75	85·09
		1900·885	32°62	84·96

A and E.

Nov.	3	1900·842	323°28	74''49
	5	·847	323°04	...
	22	·894	323°34	74·35
	26	·905	323°16	74·63
Dec.	8	·937	323°22	74·59
		1900·885	323°21	74·51

On November 5 clouds prevented distance measures. November 22 the measures were hurriedly made.

Following are the means of the different seasons' measures of the *Nova* and these two stars. Those of 1892, 1893, and 1894 were made with the 36-inch of the Lick Observatory. Those of 1900 were made with the 40-inch of this observatory.

Means for each Season of Observation.

A and E (E = 12^m).

1892·9	323°38	74''58	16 nights.
1893·9	323°43	74°51	14 "
1894·9	323°41	74°66	3 "
1900·9	323°21	74°51	5 and 4 "
1895·6	323°377	74°543	Weighting by number of nights

A and F (F = 10·5^m).

1892·9	32°63	84''94	18 nights.
1893·9	32°89	84°91	8 "
1894·9	32°63	84°94	3 "
1900·9	32°62	84°96	5 and 4 "
1895·6	32°675	84°927	Weighting by number of nights.

If now we compare the means for each year with the means of all the measures of the different years, we shall have the following :

Yearly Means Compared with the Means of all the Measures.

A and F.			A and E.		
1892	+0°045	-0''009	1892	-0°003	-0''037
1893	-0°145	+0°038	1893	-0°053	+0°037
1894	+0°045	-0°013	1894	-0°033	-0°114
1900	+0°055	-0°036	1900	+0°167	+0°031

These are in satisfactory agreement and show no certain change in the position of the *Nova*. They also show, as has been found in other measures, especially in those of the fifth satellite of *Jupiter*, that the same observer using the 36-inch of the Lick Observatory and the 40-inch of the Yerkes Observatory experiences no personality due to the different instruments, the measures made with one instrument being in perfect harmony with those made with the other.

The measures made by Mr. Burnham and given in *Monthly Notices* for 1892 April are, after correcting an error in the mean for the distance AE from 74°24 to 74''32 :

Mr. Burnham's Measures of the Nova.

A and E.			
1892°14	323°60	74''32	3 nights.
A and F.			
1892°12	32°38	85''05	4 nights.

These give the following differences from my observations :

A and E.		A and F.	
-0°022	+0''22	+0°29	-0''12

These are, considering the faintness of the objects and the distances measured, in as close agreement as would be expected from different observers. It is evident that further measures of this object for many years to come would be useless.

In looking up references to *Nova Aurigæ* the following corrections have been noted.

In *Astronomische Nachrichten* 3238, p. 391, F. Renz gives some measures of these objects. His comparison stars were E and F. In his measures the star F is put in the wrong quadrant with reference to the *Nova*, the position angle being given 212. It should be 32°.

In *Monthly Notices* for 1892 April the scale given at the bottom of Mr. Burnham's diagram is wrong. On this scale

the figures read 0'', 40'', 80''. To correspond with the diagram they should be 0'', 80'', 160''. In the same paper, at p. 436, a reference is made to *Astronomische Nachrichten* 3070: this is a typographical error for 3076.

Yerkes Observatory, Williams Bay, Wisconsin:
1901 July 29.

Further Observations of Nova Aurigæ in 1901.

By E. E. Barnard.

Nova Aurigæ has been observed on two nights this year, and on each occasion its light was estimated in terms of the comparison stars used by Mr. Burnham in 1892. The comparisons were made with the stars B, E, *e*, G, H and J.

The result from all these made the *Nova* 12.7 magnitude on this scale.

Direct estimates on the two dates made the magnitude 13.0 and 12.9 respectively.

I also made independent estimates of the magnitudes of the comparison stars. They are:

	B	C	E	<i>e</i>	F	G	H	J	M	<i>m</i>
Oct. 21	13.5	15.0	11.8	13.2	10.0	11.7	12.0	12.0	12.0	12.0
22	—	15.0	12.1	14.0	10.0	12.0	12.3	12.4	12.5	—
	13.5	15.0	11.9	13.7	10.0	11.8	12.1	12.2	12.2	12.0

Of these stars Mr. Burnham has given the following magnitudes:

B 14.8	C 15.2	E 11.7	<i>e</i> 13.2	F 10.4	G 11.5	H 11.8	J 12.7
			M 13.2	<i>m</i> 13.8.			

His estimate of the brightness of B was doubtless influenced by the proximity of the greater brightness of the *Nova* in the spring of 1892.

In my estimates of the magnitudes of M and *m* no note was made for their relative brightness. My recollection is that M was somewhat the brighter of the two.

Under the best conditions the *Nova* has not appeared well-defined when in the best focus. There is a slight haziness about it suggestive of a slightly nebulous star.

1901 October 29.

On the Variation of T Centauri. By Alex. W. Roberts, D.Sc.

The variation of the star *T Centauri* (Chandler, 4896), R.A. $13^h 36^m 2^s$ (1900), Dec.— $33^\circ 5'5$, was discovered in 1894 by Colonel Markwick, and independently the same year by Professor Pickering.

The Harvard notification of its discovery is given in the *Astronomische Nachrichten*, No. 3269, p. 72, and that of Colonel Markwick in the *Journal of the British Astronomical Association*, vol. v., p. 247. As far back as 1890, however, its variation was suspected by Kapteyn from an examination of the Cape Photographic *Durchmusterung* plates. His note on the variable magnitude indicated by the Cape Photographs is contained in *Astronomische Nachrichten*, No. 2987.

The following papers on the variation of *T Centauri* have already appeared in the *Monthly Notices*—

- Vol. lvi., p. 35 (Markwick)
- „ lvi., p. 347 (Roberts)
- „ lvi., p. 500 (Roberts)
- „ lviii., p. 513 (Markwick)

in which the nature and period of the star's variation are set forth.

Three years have now elapsed since the last of the four foregoing papers was published, and although observations made during these years have not in any way altered the general conclusions come to then, I think sufficient data have accumulated to enable a very accurate determination of the chief elements of the star's variation to be made.

In arriving at a determination of the period of *T Centauri*, or of any other long-period variable, it should be remembered that the possibility of the period being subject to a secular inequality is not unlikely. Because of this it is as well to include in the investigation only such maxima and minima as have been determined from a full-light curve, or at least from observations so situated on the light curve that they indicate, within a certain margin of error, the date of maximum or minimum brightness.

I have in other papers called attention to the uncertainty which always attaches to isolated meridian observations of magnitude; they are indeed of very little value as an aid to a definitive determination of light period unless the interval which separates the several dates is sufficiently great to compensate for the uncertainty in brightness.

If, of course, a meridian observation is manifestly taken near a maximum or minimum phase, then its value, as an aid to a

determination of the period of the star, is considerably enhanced ; but even in this case, the range in brightness is usually so irregular, amounting in the case of *T Centauri* to two magnitudes, that there is always some slight uncertainty with regard to a maximum or minimum date estimated from meridian observations alone.

I propose therefore in the present investigation to base a determination of the period of *T Centauri* upon : (1) Recent systematic observations of magnitude ; (2) All available data.

The first portion of the investigation will yield elements which satisfy present observations. If the period of the star is subject to secular inequality, these elements may not satisfy more remote observations.

In the following two series, Tables I. and II., are given the various maxima and minima determined rigorously during the past six years.

All the Lovedale observations were made with a 1-inch telescope, except where the star was passing through its maximum phase. Its brightness then allowed naked-eye estimates to be made.

TABLE I.
Determinations of Maxima.

Rotation No.	Date.	Julian Day. J	Mag. m	Observer.
1	1895 June 8	2413353	5.3	Markwick
2	June 10	2413355	5.8	Roberts
3	Sept. 10	2413447	6.0	"
4	1896 March 8	2413627		Tebbutt
5	March 10	2413629	5.4	Roberts
6	June 16	2413727	6.0	"
7	June 18	2413729	6.4	Markwick
8	Sept. 3	2413806	5.7	Roberts
9	1897 June 1	2414077	6.2	Markwick
10	1898 June 3	2414444	6.6	"
11	Sept. 1	2414534	5.9	Roberts
12	1899 Feb. 25	2414711	5.2	"
13	June 3	2414809	6.2	"
14	Aug. 28	2414895	5.5	"
15	1900 Feb. 25	2415076	6.0	"
16	May 24	2415164	5.6	"
17	Aug. 25	2415257	6.1	"
18	1901 May 23	2415528	5.9	"
19	Aug. 22	2415619	5.8	"

TABLE II.

Determinations of Minima.

Rotation No.	Date.	Julian Day. J	Mag. m	Observer.
1	1895 April 26	2413310	9.0	Roberts
2	April 27	2413311	10.0	Markwick
3	July 30	2413405	8.0	Roberts
4	1896 April 26	2413676	8.9	„
5	April 30	2413680	9.9	Markwick
6	July 24	2413765	8.7	Roberts
7	1898 April 18	2414398	9.2	Markwick
8	1899 April 10	2414755	8.9	Roberts
9	July 13	2414849	8.6	„
10	1900 April 12	2415122	7.4	„
11	July 13	2415214	8.3	„
12	1901 April 10	2415485	7.4	„
13	July 12	2415578	8.4	„

Let now the following values be considered :

$P + \Delta p$ = period of variation ;

$T_M + \Delta t_M$ = epoch of maximum ;

$T_m + \Delta t_m$ = epoch of minimum ;

m = annual amount of secular variation of period ;

E = number of periods, reckoning from the epoch ;

J = Julian day of observation as given in column 3 of Tables I. and II.

Then it is evident that the following equations represent the relation of the various quantities to one another :

With regard to Table I. :

$$(T_M + \Delta t_M) + E\{P + \Delta p\} + \frac{E}{2}m = J$$

that is,

$$\Delta t_M + E\Delta p + \frac{E^2}{2}m = \{J - (T_M + EP)\} \dots \dots (1)$$

and with regard to Table II.,

$$\Delta t_m + E\Delta p + \frac{E^2}{2}m = \{J - (T_m + Ep)\} \dots \dots (2)$$

We readily obtain from Tables I. and II. the following approximate values of T_M , T_m and P ,

$$T_M = 2415075^J,$$

$$T_m = 2415033^J,$$

$$P = 90^d.4.$$

With these assumed values and the values of J contained in column 3, we form equations of condition of the type indicated in (1) and (2).

It is not necessary to burden the paper with the statement of these thirty-two equations, or with the several steps in their solution.

The final result is as follows :

$$\begin{aligned} \Delta t_M &= +1.1^d \\ \Delta t_m &= -1.0 \\ \Delta p &= +0.07 \\ \Delta m &= 0.000 \end{aligned}$$

This gives as the final elements of *T Centauri* for the year 1901 :

$$\begin{aligned} \text{Epoch of maximum} &= 2415076.1 \\ \text{Epoch of minimum} &= 2415032.0 \\ M-m &= 44^d.1 \\ \text{Period} &= 90^d.47 \end{aligned}$$

The value of m obtained from the equations indicates that for the six years 1895-1901 the period of *T Centauri* was constant. We must not however infer from this that the period is always constant. This does not follow.

With the above corrected elements we now compute theoretical dates for comparison with the observed dates. This comparison is given in the following tables. Column 2 gives the values of E at the date of observation, column 3 the observed date ; column 4 the computed date, and column 5 the residual ($O-C$).

TABLE III.
Maxima Dates.

Rotation No.	E .	Julian Day, J	Computed Max. J	$O-C$. d
1	-19	2413353	2413357.2	-4.2
2	19	2413355	2413357.2	-2.2
3	18	2413447	2413447.6	-0.6
4	16	2413627	2413628.6	-1.6
5	16	2413629	2413628.6	+0.4
6	15	2413727	2413719.1	+7.9

Rotation No.	E.	Julian Day. J	Computed Max. J	O-C. d
7	- 15	2413729	2413719.1	+9.9
8	14	2413806	2413809.5	-3.5
9	11	2414077	2414080.9	-3.9
10	7	2414444	2414442.8	+1.2
11	6	2414534	2414533.3	+0.7
12	4	2414711	2414714.2	-3.2
13	3	2414809	2414804.7	+4.3
14	- 2	2414895	2414895.2	-0.2
15	0	2415076	2415076.1	-0.1
16	+ 1	2415164	2415166.6	-2.6
17	2	2415257	2415257.0	0.0
18	5	2415528	2415528.5	-0.5
19	+ 6	2415619	2415618.9	+0.1

TABLE IV.

Minima Dates.

Rotation No.	E.	Julian Day. J	Computed Min. J	O-C. d
1	-19	2413310	2413313.1	-3.1
2	19	2413311	2413313.1	-2.1
3	18	2413405	2413403.5	+1.5
4	15	2413676	2413674.9	+1.1
5	15	2413680	2413674.9	+5.1
6	14	2413765	2413765.4	-0.4
7	7	2414398	2413898.7	-0.7
8	3	2414755	2413760.6	-5.6
9	- 2	2414849	2414851.1	-2.1
10	+ 1	2415122	2415122.5	-0.5
11	2	2415214	2415212.9	+1.1
12	5	2415485	2415484.4	+0.6
13	+ 6	2415578	2415574.8	+3.2

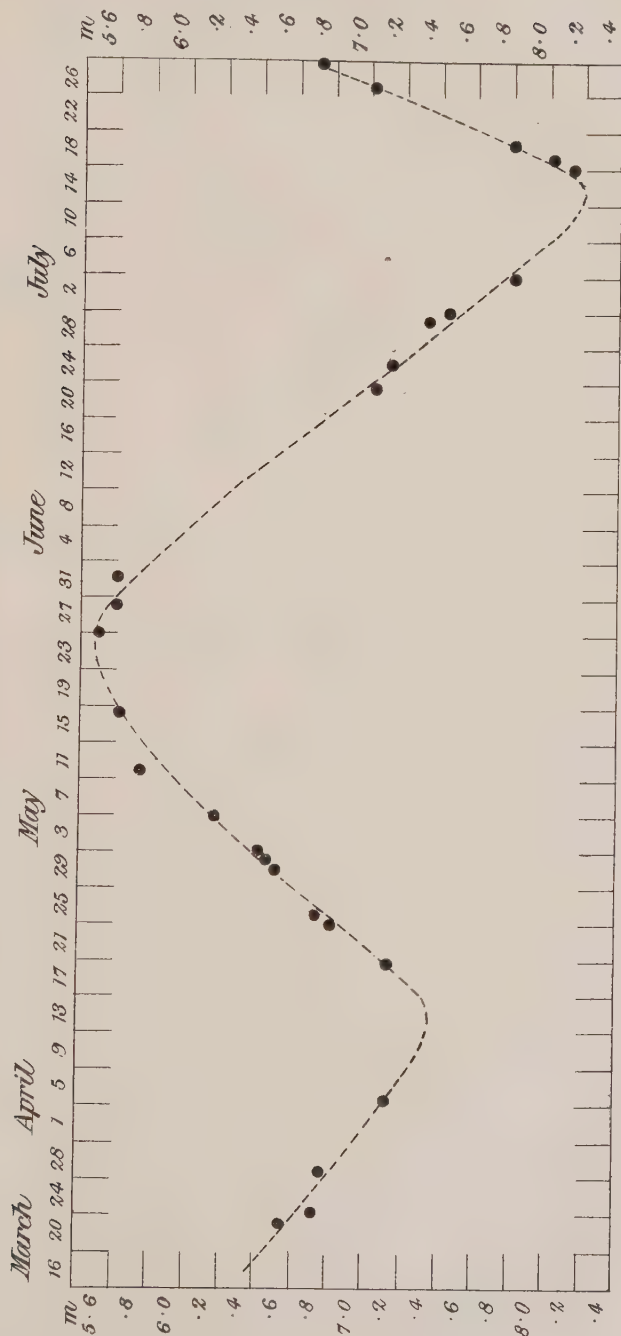
The average residual is :

				d.
Maxima dates	...	...	...	2.5
Minima dates	...	...	...	2.1

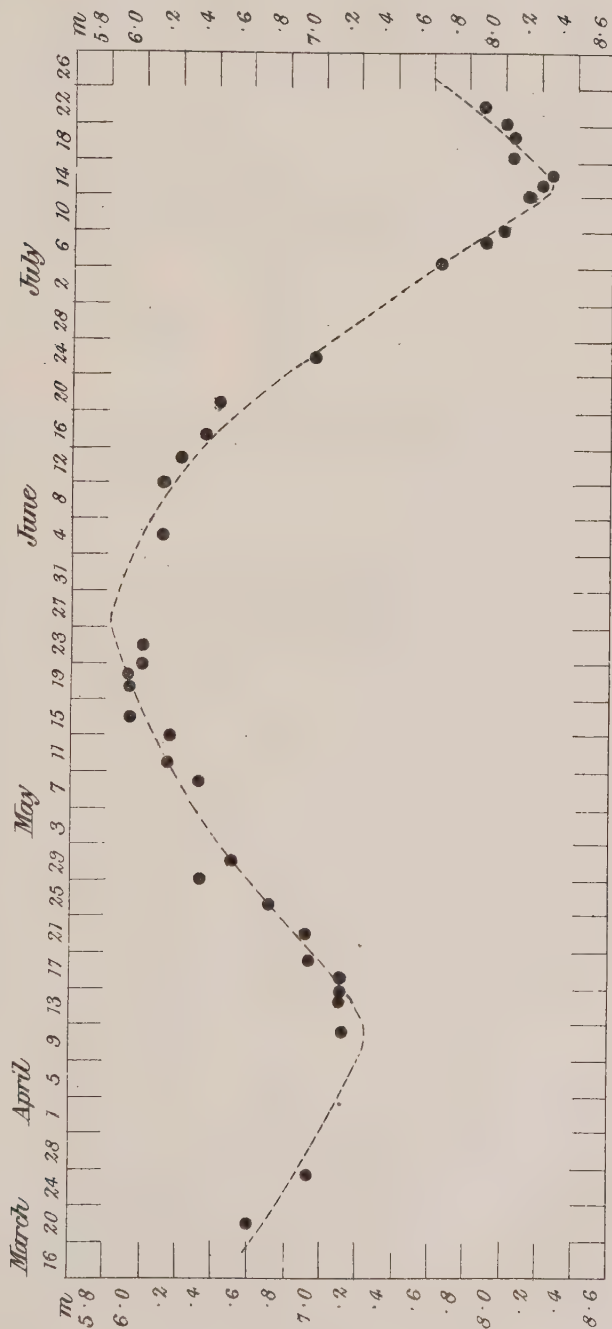
That the latter value is the smaller is due to the fact that the minimum of this star is much more distinctly defined than the maximum phase.

The important element

$$M - m = 44^{\text{d}}.1$$

Light Curve of γ Centauri, March to July 1900.

Light Curve of γ Centauri, March to July 1901.



agrees very closely with that determined by Colonel Markwick, forty-three days (*Monthly Notices, R.A.S.*, vol. lviii., p. 514), and indicates that the duration of the ascending period of variation is not markedly unequal to the descending period.

This practical equality between ascending and descending portions of the light curve is a feature of not a few long-period stars.

Colonel Markwick finds a lower limit of $10^{\text{m} \cdot 0}$ for the variation of *T Centauri*, but I think this is due to his underestimate by more than a whole magnitude of the brightness of two of the comparison stars used by him. The two stars are ;

		h	m	s	°	'	
Z.C. xiii. ^h	2039	13	34	0	33	1'3	(1875)
	2082	13	34	36	33	7'9	

The mean brightness of both stars is given in the Zone Catalogue as $8\frac{3}{4}$ magnitude. They have been regularly used at Lovedale as comparison stars for *T Centauri*, and I estimate them as between 8.6 and 8.8 magnitude. They are both distinctly seen in a 1-inch glass.

Colonel Markwick however considers them as $10^{\text{m} \cdot 0}$ magnitude, a value which I am convinced is $11\frac{1}{2}$ magnitude too faint.

If allowance be made for this disparity in the reference standards it will be found that the range of magnitude obtained by Markwick confirms that determined at Lovedale.

In one of his papers on this variable Colonel Markwick pointed out that the range of variation was subject to considerable irregularity. This irregularity can be well exemplified by charting down two full cycles, one cycle in 1900 and the other in 1901.

These light curves are given in plates 1 and 2. The apparent similarity between the curves, indicating apparently alternative primary and secondary minima, is fortuitous. There is no regular succession of bright and faint minima, as is the case in some of the better known long period variables.

What has been stated or determined in the preceding part of this paper is deduced directly and solely from recent observations. In the case of *T Centauri* we have fortunately some old observations which I think may reasonably be used either to correct or supplement results obtained from more recent data.

These observations are :—

	Mag.	
1752 June 2	7.0	Lacaille
1846 June 6	8.0	Washington Obsns.
1849 Feb. 13	8.9	"
March 23	8.0	"
1862 April 17	6.0	Yarnall
1863 April 17	6.0	"

		Mag.	
1870	Jan. 7	6.0	Yarnall.
	Jan. 17	6.0	"
1872	May 4	7.5	Washington Obsns.
	May 14		
	May 27		
1873	June 17	7.0	Cordoba Obsns.
	June 30	7.5	"
1874	July 2	6.8	"
	July 3	7.5	"
	July 4	7.3	"
	July 11	7.0	"
1875	June 16	7.0	"

As an indication of the uncertainty which must belong to any determination based simply on meridian observations of magnitude, Colonel Markwick regards Lacaille's observation as taken near a maximum of the star. In my judgment the observation was taken near a minimum. In such uncertainty as this the observation must be set aside.

With regard to the early Washington observations we have more definite data to go upon. It is evident that a minimum took place on or about 1849 February 13. Yarnall's observations, also, clearly point to a maximum on or about 1862 April 17.

The Cordoba observations do not indicate a point of reference that we may accept with confidence. I think, however, that 1874 June 15 as the date of a maximum phase is not much in error.

In the *Astronomische Nachrichten*, No. 3269, Professor Pickering gives a valuable series of photographic magnitudes of *T Centauri*.

These indicate a minimum on 1893 July 27.

We have therefore the following four dates to which we may relate recent observations :—

1849 Feb. 13	Minimum	Julian Day. 2396437
1862 April 17	Maximum	2401248
1874 June 15	"	2405690
1893 July 27	Minimum	2412672

It is found that a period of $90^{\text{d}}.30$ will better satisfy the earlier measures than the period $90^{\text{d}}.47$ determined from recent observations. The correspondence with this shorter period is as follows :—

E.	Observed date.	Computed date.	O-C. d.
- 206	2396437	2396430	+ 7
153	2401248	2401260	- 12
104	2405690	2405685	+ 5
- 26	2412672	2412684	- 12

If the larger value, $90^{\text{d}}.47$, be related to the last observation the residual is only -7^{d} . It is evident, therefore, that the period of *T Centauri* is affected by some secular variation, the nature and amount of which we can only discover when a fuller series of observations has been secured.

The mean period of the star will probably be found to be not far from 90.3 days, with a range of from two to three days on either side of this mean period.

I would seek to acknowledge my great indebtedness to Sir David Gill and to Mr. R. T. A. Innes for their ever-ready willingness to make remoteness from a complete astronomical library less of a hindrance in investigations such as the preceding. They were good enough in the present instance to look up for me all the old records of *T Centauri*.

Lovedale: 1901 October 1.

On the Place of the Variable RU Herculis and Neighbouring Stars from Photographic Measures. By F. A. Bellamy, F.R.Met.Soc.

In No. 3384 of the *Astronomische Nachrichten* Dr. T. D. Anderson refers to a star of the ninth magnitude not given in B.D. He noted it as 9.1 magnitude in 1895 August; it was below the tenth magnitude on October 16, and became invisible later.

As there were several stars in the field of view of similar magnitudes, which rendered it difficult to identify the probable variable star with certainty, it seemed necessary that the position of it should be accurately determined, and this Dr. Hartwig did by measurement from two stars in the Cambridge and Berlin A.G. Catalogues with the heliometer; his results are included in an article on variable stars in *Astronomische Nachrichten*, 3553, and the positions given are

$$1900.0 \text{ R.A. } 16^{\text{h}} 6^{\text{m}} 2^{\text{s}}.76 \quad + 25^{\circ} 19' 54''.7$$

and he adds a note to the effect that his observation was rendered difficult by the low altitude of the star and atmospheric conditions.

This star is in the region apportioned to this observatory for

the Astrographic Chart, and upon examination we find that it has been photographed on three plates.

	h m				
427 R.A.	16 4	+25°	exposed	1893 June 12	
778	16 8	+26		1895 May 29	
1406	16 8	+26		1899 May 26	

Plate 1406 replaces plate 778, as the latter only showed 76 stars with 2 images and 11 stars with 3 images, the exposures being 6^m, 3^m, and 20^s. Plate 1406 has 188 and 80 stars.

It seems important, as pointed out in a previous paper on *Nova Persei*, that the positions of variables and neighbouring stars should be given accurately. The object of this paper is to give accurate places of the variable star *RU Herculis*, and of all the stars photographed within 20' of it; therefore all stars probably to the eleventh magnitude. By means of these the positions of other intermediate and fainter stars can readily be determined when required for comparison.

The measures of plate 427 were made by Mr. H. A. Freeborn in 1897 July 23, and of plate 1406 by Miss E. F. Bellamy 1900 September 11-14. The constants *a*, *b*, *c*, *d*, *e*, *f* were determined from a comparison of the computed ξ' and η' (1900·0) for stars in the Cambridge and Berlin A.G. Catalogues, with the measures corresponding to the same stars; they are for

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>
Plate 427	+·00762	+·00164	—·3719	—·00190	+·00762	—·0918
1406	—·00026	+·00447	—·2910	—·00436	—·00045	—·1314

The application of these constants in the usual way gives the ξ' and η' in Table I., but these coordinates need only be given for plate 1406; the right ascensions and declinations (1900·0) were obtained for both plates, and the positions are the means of the two. The eighth and ninth columns contain the differences between the two plates.

The magnitudes in the third column are estimations, to the nearest third or half magnitude, made by myself in November this year.

TABLE I.

Ref. No.	Camb. A.G.C.	Oxford Mag.	ξ' 1900·0	η' 1900·0	Deduced				Difference 1406-427	
			Plate 1406.	1406.	R.A. 1900·0.	N. Dec. 1900·0.			in R.A.	in Dec.
					h m s	°	'	''	s	"
1	7492	9·5	1·3834	2·9930	16 3 43·30	25	9	44·6	+0·13	—0·7
2		10·0	2·6205	1·9904	4 10·78		4	47·0	+·12	—1·3
3		9·7	2·7987	4·5099	4 14·32		17	23·5	+·11	—2·2
4		11·0	2·8255	6·4584	4 14·66		27	6·8	...	...
5		11·0	2·9175	8·9776	4 16·30		39	42·6	...	...

Ref. No.	Camb. A.G.C.	Oxford Mag.	ξ'	η'	Deducted		Difference	
			1900'0 Plate	1900'0. 1406.	R.A. 1900'0.	N. Dec. 1900'0.	in R.A.	in Dec.
6		10'0	2'9761	5'4301	16 ^h 4 ^m 18 ^s ·17	25° 21' 59"·1	+ 0'06	- 0'8
7		10'5	3'5413	2'9689	4 30'94	9 41'8	+ '16	- 0'2
8		10'0	3'7853	8'7807	4 35'57	38 45'2	+ '01	+ 0'2
9		9'7	3'7778	6'5631	4 35'71	27 40'4	+ '04	- 0'6
10	7496	8'5	3'9927	3'3620	4 40'90	11 40'5	+ '11	- 0'1
11		9'7	4'7037	6'5747	4 56'36	27 45'4	+ '01	- 0'1
12		10'3	4'7900	6'8087	4 58'11	28 56'0	'00	- 0'7
13		10'7	6'9414	4'3294	5 45'93	16 35'8	+ '15	- 1'3
14		10'0	7'3974	3'1658	5 56'15	10 47'0	+ '08	- 0'5
15		11'0	7'5099	6'2204	5 58'41	26 3'1	...	...
16	Variable		7'7077	4'9903	6 2'89	19 54'3	...	...
17		10'3	7'8055	3'3878	6 5'15	11 53'8	+ '06	0'0
18		11'0	8'1264	2'4336	6 12'24	7 8'3	+ '20	- 0'9
19		11'0	8'5363	5'6330	6 21'18	23 7'9	...	...
20		9'5	8'6046	7'2410	6 22'58	31 10'9	- '01	- 1'2
21		10'0	8'7604	6'4043	6 26'09	26 59'6	'00	- 0'2
22		10'7	8'8989	2'2604	6 29'35	6 16'6	+ '15	+ 0'3
23		9'5	9'2947	7'5711	6 37'88	32 50'8	- '04	- 1'6
24		11'5	9'6254	5'2190	6 45'31	21 4'7	...	...
25		11'5	9'8601	1'9414	6 50'66	4 41'7	...	...
26		10'0	10'2265	6'6907	6 58'54	28 26'5	+ '02	- 0'3
27		11'3	10'3339	8'3776	7 0'86	36 52'6	...	...
28		9'5	10'7712	6'9873	7 10'62	29 56'1	- '02	- 0'8
29	7525	8'7	10'8004	3'7127	7 11'38	13 33'9	- '05	+ 0'4
30	7528	8'5	10'9892	9'1522	7 15'38	40 46'2	- '04	- 1'9
31	7533	8'7	12'1575	6'9210	16 7 41'32	25 29 36'3	'00	0'0

Nos. 4, 5, 15, 19, 24, 25, 27 are not visible on plate 427. Though *RU Herculis* is visible on plates 427 and 778, the images are so faint that satisfactory measures cannot be made; when the scale is placed over them they become invisible.

With regard to the differences (especially in R.A.) it should be mentioned that the constants for these two plates (427 and 1406) were derived from stars in the Catalogue of the Ast. Gesellschaft, observed at Cambridge and Berlin. For plate 1406 all the stars are from the Cambridge, whilst for plate 427 five of

the twenty-five stars used are from the Berlin B. Catalogue, all these being in the southern half of the plate. The question arises whether there is any systematic difference between the R.A.'s of Cambridge and Berlin B. Catalogues.

In the introduction to the Greenwich 1890·0 Catalogue, p. 47, comparisons are given Greenwich—Cambridge and Greenwich—Berlin B., and the systematic differences in R.A. are $-0^s\cdot04$ (198 stars) and $-0^s\cdot03$ (282 stars), whence C—B is $+0^s\cdot01$. But this comparison depends on the brighter stars only, and for our present purpose is illusory; for, the majority of stars in Cambridge and Berlin B. Catalogues are of the 8th to the 9th magnitude, and even fainter, and the Cambridge observations are affected by a very pronounced magnitude equation, as pointed out by Mr. A. R. Hinks (*Monthly Notices*, lvii., p. 473), using 671 stars, and by Professor Turner (*Monthly Notices*, lx., p. 3), using 5,384 stars. In fact, while the R.A.'s of the mean bright star (probably about the $5\frac{1}{2}$ or 6th magnitude) of the Cambridge and Berlin are the same, those of the mean 8th and 9th magnitude star differ by $0^s\cdot07$ at least.

If, then, the places from Cambridge and Berlin are taken just as they stand, the lower and southern half of the plate, depending chiefly on Berlin stars, will be displaced in R.A. with reference to the northern half of the plate; and this will alter the orientation (constant b) deduced from the x coordinates. Assuming the above difference of $0^s\cdot07$, the effect is to increase b by $+0^m\cdot00025$, and there is a consequent correction of $-0^m\cdot0053$ to c . A systematic difference has been noticed in declination by Mr. Hinks, the amount being $0''\cdot5$, which will similarly alter the scale value (constant e) deduced from the y coordinate.

In the particular case of plate 427 these quantities are probably excessive, for there are 5 Berlin stars in the lower half, but the total number is 8. Nevertheless they are quite inadequate to account for the systematic difference in R.A. as appears from the subjoined Table II., in which the differences in Table I. are compared with those due to the above alterations in b and c .

The main cause of the discordance between the two plates must accordingly be sought for elsewhere—probably in the accidental errors of the reference stars (including proper motions from the dates of meridian observations to 1900·0, and for the six years between the two plates)—and though these will ultimately be fully discussed, for the present it will serve the main purpose of this paper to give the mean of the results as they stand, as has been done in Table I.

TABLE II.

Plate 1406-427.

Ref. No.	Difference as in Table I. s	Correction for Magnitude Equation. s	Difference Corrected. s
1	$+0^s\cdot13$	$-0^s\cdot03$	$+0^s\cdot10$
2	$+0^s\cdot12$	$-0^s\cdot03$	$+0^s\cdot09$

Ref. No.	Difference as in Table I. s	Correction for Magnitude Equation. s	Difference Corrected. s
3	+ 0.11	- 0.02	+ 0.09
6	+ .06	- .01	+ .05
7	+ .16	- .03	+ .13
8	+ .01	+ .01	+ .02
9	+ .04	- .01	+ .03
10	+ .11	- .03	+ .08
11	+ .01	- .01	00
12	.00	- .01	- .01
13	+ .15	- .02	+ .13
14	+ .08	- .03	+ .05
17	+ .06	- .03	+ .03
18	+ .20	- .03	+ .17
20	- .01	.00	- .01
21	00	- .01	- .01
22	+ .15	- .03	+ .12
23	- .04	.00	- .04
26	+ .02	.00	+ .02
28	- .02	.00	- .02
29	- .05	- .02	- .07
30	- .04	+ .01	- .03
31	.00	.00	.00

The plates afford some information as to the magnitude of the variable. On plate 427 ($16^h 4^m + 25^\circ$ 1893 June 12) it is at the limit of visibility; in fact, it had been omitted in the original measures of the plate, though stars certainly as faint as 10.5 mag. had been measured, and was only detected when examining it for this paper by first computing its coordinates, and so knowing exactly where to look for it.

The star is visible on plate 778 ($16^h 8^m + 26^\circ$ 1895 May 29), but again was only found by previously ascertaining its exact position. The plate had been rejected as having an insufficient number of stars; there were only 11 stars shown by the 20^s exposure, whereas Argelander gives 57 in the same region. It can be inferred that quite 1.5 magnitudes were lost on this plate, and so the magnitude of *RU Herculis* would be about 9.5. It was in 1895 August that Dr. Anderson first noticed that the star was missing in the B.D., and he then estimated it to be 9.1.

When the third plate (1406) was taken, 1899 May 26, the star was very near its maximum, and was undoubtedly as bright or brighter than Camb. 7533 (B.D. + 25° , 3046), magnitude 9.0.

If the star is red in appearance this will be equivalent to increasing the mags. given from the plates by at least several tenths

Dr. Hartwig gives the period as 473* days. According to Mr. J. A. Parkhurst, the star is equal to the fourteenth magnitude at its minimum.†

Further Observations of the New Star in Perseus made at the Radcliffe Observatory, Oxford.

(Communicated by Arthur A. Rambaut, M.A., Sc.D., F.R.S.,
Radcliffe Observer.)

This paper contains the results of observations upon the brightness and colour of the new star in *Perseus* made at the Radcliffe Observatory since the date of the last meeting of the Society, and is in continuation of the Notes on the same subject already communicated to the Society, and published in the *Monthly Notices* for March, April, May, and June.

The periodical fluctuations in the brightness of the *Nova* which had been observed up to the end of the first week in June appeared to die out about the middle of that month. The last observed maximum ($4^m.57$) was on June 8. The next estimation of magnitude was $5^m.80$ on June 17, and since that date the observations show a gradual diminution of brightness at the rate of about $0^m.01$ per diem, the magnitude on November 1 being 7.1 . If the magnitudes be plotted as ordinates with the dates as abscissæ, a straight line drawn through them will lie within $0^m.2$ of all observations, and the discrepancies show no trace of periodicity. Small changes of brightness have occasionally been suspected by the observers in the course of a night's observations.

During the months of June and July the star was observed under unfavourable conditions at a small altitude above the northern horizon. In July the image of the *Nova* was frequently noted as being "dull," and "larger" than the other stars, but since then it has generally appeared sharp and stellar.

On August 22 the image of the *Nova* was carefully examined to test whether any trace could be found visually of the aureole which MM. Antoniadi and Flammarion had seen surrounding the photographic image. No indication of such an aureole could be discovered (see Observers' Notes for that date).

A marked change has also taken place in the colour of the star. The redness previously conspicuous at minimum has gradually disappeared, and the star is now for the most part colourless. On June 17 it was described as very red. From August 8 to August 24 it appeared pale yellow with occasionally a red fringe which did not show in the other star images. From August 27 to November 1 it is generally noted as colourless, with,

* *Vierteljahrsschrift der Astron. Gesell.*, Jahr. 34, p. 315.

† *Astrophysical Journal*, vol. xiv. p. 172.

on one or two occasions, an indication of blue. Since July 3 the star *Arg. Z.* +43°, 730 (No. 58 in Table I.) has appeared considerably redder than the *Nova*.

The stars *Arg. Z.* +44°, 734 and *Arg. Z.* +43°, 730 (Nos. 54 and 58 in Table I.) have been frequently used as comparison stars. If the magnitudes given in the Harvard Photometry, Harvard Photometric Revis., and Harvard Photometric D.M., be accepted, it is found that the magnitude of the *Nova* resulting from a comparison with these stars differs systematically from that deduced by comparison with the other stars. Comparing the magnitudes of these two stars with a number of others the resulting magnitudes on the Harvard scale were found to be 6.23 and 6.57 respectively. The magnitudes given for No. 54 in the H.P., H.P. Revis., and H.P. D.M. are 6.04, 6.32, and 6.45. The magnitude given for No. 58 in the H.P. D.M. is 6.91. It will be seen that the Radcliffe estimation of No. 54 on the Harvard scale agrees more closely with the mean of the three photometric results (6.27) than with the first of them, the magnitude hitherto adopted.

These revised magnitudes have been used in Table I. and throughout the comparisons given in this paper.

The following instruments were employed as indicated in the fourth column of Table II. :—

<i>Instruments Used.</i>		
Barclay	10-inch equatorial	Power 90
	2.7-inch finder	,, 13
Heliumeter	7.5-inch	,, 45
	1.8-inch finder	,, 19
Ramsden	2.2-inch	,, 40
Naked eye.		

TABLE I.

List of Stars used for Comparison with Nova Persei.

Ref. No.	Name of Star.	Harvard Photom. Mag.	Ref. No.	Name of Star.	Harvard Photom. Mag.
50	<i>l</i> Persei	4.84	61	<i>Arg. Z.</i> +41°, 631	6.00†
51	31 Persei	4.99	62	<i>Arg. Z.</i> +41°, 638	6.16†
52	30 Persei	5.37	63	<i>Arg. Z.</i> +42°, 736	7.00§
53	36 Persei	5.40	64	<i>Arg. Z.</i> +43°, 720	7.25§
54	<i>Arg. Z.</i> +44°, 734	6.23*	65	<i>Arg. Z.</i> +43°, 732	7.25†
55	<i>Arg. Z.</i> +44°, 648	6.47†	66	<i>Arg. Z.</i> +47°, 847	5.52†
56	<i>Arg. Z.</i> +43°, 818	5.97	67	<i>Arg. Z.</i> +44°, 714	7.37†
58	<i>Arg. Z.</i> +43°, 730	6.57*	68	<i>Arg. Z.</i> +47°, 844	6.04†
59	<i>Arg. Z.</i> +42°, 795	6.30†	69	<i>Arg. Z.</i> +47°, 846	6.77†
60	<i>Arg. Z.</i> +44°, 656	7.60§	70	<i>Arg. Z.</i> +44°, 732	7.32†

* Radcliffe estimations on Harvard scale.

† Harvard Photom. Revision.

† Harvard Photom. D.M.

§ Adopted provisionally.

TABLE II.

Means of Estimations of Magnitude of Nova Persei.

	G.M.T.	Observer.	Aperture of Telescope. inch	Reference Stars.	Mean Mag.	Adopted Mag. for the night.
1901. June 17	h m 10 40	R.	7.5	52, 56	5.80	5.80
	18 10 50	R.	7.5	52, 54, 56	5.99	5.99
July 1	11 5	R.	7.5	52	5.90	5.90
	3 11 0	R.	7.5	52, 54	6.00	6.00
	8 11 30	R.	7.5	52	5.97	5.97
	9 11 45	R.	1.8	52, 56, 54, 58	5.91	6.07
	11 45	R.	7.5	52, 54, 58	6.22	
15	10 30	R.	1.8	54, 52, 56, 58	6.02	6.11
	10 30	R.	7.5	54, 52, 58	6.16	
	11 40	A. A. R.	7.5	58, 54	6.15	
	11 40	A. A. R.	1.8	58, 54	6.15	
	16 10 15	R.	1.8	52, 56, 54	5.86	5.86
	17 11 0	R.	1.8	52, 56, 54, 58	6.22	6.22
	18 11 0	R.	1.8	56, 52, 54	5.99	5.92
	11 20	C.	1.8	56, 52, 54, 58	5.85	
	19 10 30	R.	1.8	52, 56, 54	5.92	5.92
	11 30	R.	2.7	52, 56, 54	5.92	
	20 10 0	R.	1.8	52, 54, 56	5.92	5.92
	22 10 30	R.	1.8	56, 52, 54	5.97	5.97
	23 10 30	R.	1.8	52, 54	5.93	5.93
	24 11 15	R.	1.8	52, 54, 56	6.09	6.09
Aug. 8	11 45	C.	2.7	52, 56, 55, 54	6.34	6.34
	10 10 30	C.	2.7	54, 52, 55, 56	6.45	6.45
	15 9 30	C.	2.7	55, 54, 58, 56	6.50	6.50

Nov. 1901.

the New Star in Perseus.

81

1901. Aug.	G.M.T.		Observer.	Aperture of Telescope, inch	Reference Stars.	Mean Mag.	Adopted Mag. for the night.
	h	m					
17	10	0	C.	2.7	56, 55, 54, 67	6.51	6.51
20	10	50	R.	Naked eye	54, 59	6.46	6.46
22	9	30	W.	2.7	54	6.13	6.22
	9	30	W.	10.0	54, 53	6.02	
	10	45	R.	Naked eye	54, 59	6.46	
24	10	20	R.	10.0	56, 54	6.30	6.30
27	11	53	R.	2.7	56, 54, 58	6.42	6.42
28	8	45	R.	1.8	58, 54	6.25	6.25
Sept. 3	7	45	R.	7.5	54, 56, 70-54, 58	6.65	6.54
	9	0	R.	Naked eye	54, 59	< 6.30	
	10	45	R.	2.7	56, 58, 54	6.42	
4	7	43	R.	7.5	54, 70	6.77	6.67
	7	43	R.	1.8	53, 70, 54, 58	6.77	
	8	45	R.	1.8	56, 54, 70, 58	6.57	
	9	30	R.	Naked eye	54, 59	6.57	
5	9	7	W.	2.7	58, 62, 61, 55, 54, 68, 69	6.46	6.45
	9	7	W.	10.0	58, 62, 61, 55, 54, 68, 69	6.44	
	10	45	R.	Naked eye	54, 59	< 6.30	
7	9	25	R.	2.7	65, 54, 58	6.58	6.58
9	9	40	W.	10.0	58, 64, 65	6.77	6.65
	9	40	W.	2.7	58, 62, 61, 64, 55, 69, 68, 66	6.72	
	10	15	R.	Naked eye	54, 59	6.62	
	11	5	W.	2.2	58, 54, 65	6.42	
10	9	35	R.	2.7	56, 54, 58, 65, 64	6.72	6.72
11	7	40	R.	7.5	70, 54, 56	6.74	6.54
	9	40	W.	2.7	58, 54, 62, 63, 56, 69	6.52	
	9	40	W.	10.0	58, 54, 62, 63	6.43	
14	8	20	W.	2.7	54, 58	6.80	6.65
	8	30	R.	7.5	54, 70, 56	6.74	
	9	30	R.	1.8	54, 56, 58, 65	6.46	

G

	G.M.T.	Observer.	Aperture of Telescope. inch.	Reference Stars.	Mean Mag.	Adopted Mag. for the night.
1901. Sept. 18	h m					
	7 50	R.	7.5	58, 54, 70, 56	6.87	6.79
	8 55	R.	1.8	65, 64, 54, 56, 58	6.71	
	21 9 30	C.	2.7	54, 65, 56	6.73	6.73
	25 8 5	R.	2.7	58, 54, 65, 64, 56, 70, 67	6.72	6.72
	26 8 30	C.	2.7	54, 65, 56, 58	6.75	6.74
	8 53	W.	2.7	58, 65, 54, 62, 63, 55, 60	6.73	
	28 9 0	C.	2.7	65, 54, 58, 64, 56, 55	6.81	6.81
Oct. 4	9 15	R.	2.7	58, 54, 70, 67, 65, 64, 56	6.84	6.84
	5 9 8	C.	2.7	65, 54, 64	6.74	6.74
	9 7 45	R.	2.7	56, 54, 70, 67, 58, 65, 64	6.84	6.84
	18 6 50	R.	2.7	65, 64, 58, 54, 70, 67	6.94	6.94
	19 8 15	C.	2.7	54, 65, 58, 67, 56	6.93	6.93
	21 7 15	R.	10.0	58, 54, 70, 56	6.79	6.79
	22 8 30	C.	2.7	65, 54, 58, 67	6.86	6.86
	24 8 38	C.	2.7	65, 64, 54, 67, 70, 58	7.16	7.16
	25 6 45	R.	2.7	65, 64, 70, 67	7.32	7.15
	6 45	R.	10.0	65, 64, 70	7.39	
	9 0	W.	1.8	58, 54	6.70	
	9 30	R.	1.8	65, 64, 67, 70, 58	7.06	
	30 11 0	R.	2.7	65, 64, 58, 67, 70	7.03	7.03
	31 8 38	C.	2.7	65, 64, 58, 67, 70	7.20	7.20
Nov. 1	7 0	R.	2.7	65, 64, 58	7.12	7.12

Observers' Notes on the Estimations of Magnitude.

1901.
 June 17. Nova low, but sky very clear.
 July 1. Sky very cloudy.
 3. Clouds passing, but estimations fairly reliable.
 8. Images varying through shifting fog-banks.
 9, 15, 17, 19. In the 7.5-inch the Nova is fainter than No. 58, whilst in the 1.8-inch finder it is much brighter. The image of the Nova is, in the 7.5 inch, duller than that of No. 58, but it is larger. (R.)

- 1901.
- July 15. No. 58 is brighter than No. 54 in the 7·5-inch, but the two stars are equal in the 1·8-inch. In both telescopes the Nova is brighter than No. 54. (A.A.R.)
18. Comparison with No. 58 difficult owing to close proximity of other stars. Sky hazy. (C.)
19. Image of Nova in the 10-inch is duller than that of No. 58. Sky hazy in places.
23. Sky nearly overcast; observed in breaks.
24. Difficult observations; cloud occasionally passing.
- Aug. 10. Sky very clear and definition good.
15. Haze and thin cloud prevalent. Definition not satisfactory.
17. Sky very clear in Perseus. Definition good.
20. Nova just visible to the naked eye, fainter than Nos. 54 and 59. Sky brilliantly clear.
22. To test whether Flammarion's and Antoniadi's photographic aureole could be detected visually, examined Nova with Barclay Equatorial. No difference of image as compared with other stars could be seen with "finder" or 10-inch telescope, using powers 90 or 250. With 250 all images were diffused to-night. (W.) Nova visible to the naked eye by glimpses only. (R.)
24. In the 10-inch No. 54 is distinctly brighter than Nova, but in the 2·7-inch "finder" it is fainter. Sky foggy.
27. Sky clear, but moonlight strong.
- Sept. 3. With the 7·5-inch images sharp and good; objects low, observed in twilight. At times the image of the Nova in the 2·7-inch seemed larger than that of No. 54, but its light was duller. With the naked eye Nos. 54 and 59 were seen with but little difficulty, but the Nova was not visible.
4. Observations difficult with the 1·8-inch. The brightness of the Nova seemed to fluctuate in short intervals to the extent of about 0·2 magnitude. With the naked eye the following sequence of magnitude was recorded:—Nos. 50; 53 and 52 nearly equal; 54 and 59 nearly equal; Nova, by glimpses, but suspected only.
5. To the naked eye Nos. 54 and 59 just visible. Nova invisible. (R.)
7. Images varying; clouds passing. Observations difficult.
9. Sky very clear. (W.) No. 54 is easily visible to the naked eye, brighter than No. 59, which is also picked up with but little difficulty. Both stars are distinctly brighter than Nova, which shows up in glimpses only. (R.)
11. Sky hazy. (R.) Thin cloudlets passing at times and occasional large tracts of cloud. Very good images generally. (W.)
14. A hurried comparison. (W.)
18. Image of the Nova sharp and stellar with the 7·5-inch. Some haze passing during observations with the 1·8-inch.
21. Clouds passing. Object-glasses of 10-inch and 2·7-inch "finder" became quickly covered with moisture. Very damp atmosphere.
25. In the 2·7-inch Nos. 54 and 58 are equal in magnitude. In the 10-inch No. 67 is double, but in the 2·7-inch single.
- Oct. 5. Hurried estimations owing to prevalence of cloud.
21. Through passing cloud and mist; observations difficult.
22. Haze and thick low fog prevalent.
25. With the 2·7-inch No. 65 is slightly fainter than No. 64; and Nova is considerably fainter than No. 58. With the 10-inch, power 90, No. 65 is fainter than No. 64. Sky hazy in places and moonlight strong. With the 1·8-inch No. 65 is slightly brighter than No. 64. Sky clearer than at 6^h 45^m G.M.T.; but some haze still prevalent, with moonlight. (R.)

Observers' Remarks on Colour of Nova and Comparison Stars.

- 1901.
- June 17. Nova very red.
- July 1. No marked colour in Nova.
 3. Nova is red, but not nearly so red as No. 58.
 9. With the 7·5-inch Nova is orange, and No. 58 orange-red.
 15. Nova orange-red, but less red than No. 58, in the 7·5-inch. (R.)
 No. 58 more red than Nova in both telescopes. (A.A.R.)
 16, 17. Nova orange; No. 58 orange-red.
 18. Nova orange and No. 58 red in the 7·5-inch. (R.) Nova not so red as No. 58 in the 7·5-inch. (C.)
 19. In both 7·5-inch and 10-inch Nova orange and No. 58 orange-red.
 22. In the 7·5-inch Nova orange-red; No. 58 red.
 24. In the 7·5-inch Nova less red than No. 58.
- Aug. 8. In the 10-inch Nova pale yellow, much less coloured than No. 58.
 10. In the 10-inch Nova much less coloured than No. 58, which is deep orange. At times suspected a reddish fringe to the Nova.
 17. In the 10-inch Nova is pale yellow; in strong contrast to No. 58, which is deep orange.
 24. Nova orange, not so deep as No. 58. Sky foggy. Moon ruddy, or deep orange.
 27. With the 10-inch, power 90: Nova white, with small red wings, and No. 58 orange. Marked contrast in colours with powers 180 and 200. Nova nearly colourless, and No. 58 red-orange. Definition not good with the latter powers to-night.
 28. In the 7·5-inch the Nova is colourless and No. 58 orange.
- Sept. 3. No colour to Nova in the 7·5-inch, except at intervals, when a merest tinge of red was suspected; No. 70 is reddish; No. 58 rich orange-red.
 4. In the 7·5-inch No. 58 orange-red, Nova colourless. In the 1·8-inch Nos. 70 and 58 both orange-red.
 5. In the 10-inch No. 62 is yellow-red, No. 61 bluish-white, and No. 55 markedly red-yellow. (W.)
 7. No. 58 orange-red in the 2·7-inch.
 9. No. 58 is red in the 2·7-inch and No. 55 is a pronounced red. In the 10-inch with power 90 no marked colour to Nova. With power 100 Nova is reddish; also reddish with power 200, but not so marked. No. 58 is strongly red. (W.)
 10. In the 10-inch No. 58 is orange-red and Nova colourless.
 11. No colour to Nova in 10-inch or 2·7-inch. (W.)
 14. Nova colourless in the 7·5-inch and No. 70 orange-red. (R)
 In the 10-inch, power 90, the image of the Nova was, at 10^h G.M.T., a bluish-white disc with a narrow reddish fringe. (W.)
 18. No. 58 orange-red in the 7·5-inch.
 25. In the 10-inch Nova is bluish-white with very small red wings. No. 58 is red-orange. No. 67 is double; the brighter star white, the fainter orange.
 26. With the 10-inch and power 90 could not detect any coloured fringe to Nova. (C.) No. 58 yellow in the 2·7-inch. With 10-inch and power 90 Nova is very light yellow, with a faint tinge of red on the border of the image. (W.)
 28. In the 10-inch, power 90, Nova is a very pale yellow, with a faint red border showing at times.
- Oct. 4. In the 10-inch, power 90, Nova bluish-white and No. 58 reddish-yellow.
 9. In the 10-inch, power 90, Nova white and No. 58 orange.
 18. In the 10-inch, power 90, Nova white; No. 58 orange; No. 67 is double, the fainter star reddish.
 21. Nos. 58 and 70 are orange-red in the 10-inch.

1901.
 Oct. 24. Nova white and No. 58 deep orange in the 10-inch.
 25. No. 64 is light orange and No. 70 orange-red in the 10-inch, power 90. (R.)
 31. In the 10-inch, power 90, Nova is white and is $\frac{1}{3}$ rd mag. fainter than No. 58, which is deep orange.

The observers were: Dr. Rambaut, indicated by A.A.R.

Mr. Wickham,	„	W.
Mr. Robinson,	„	R.
Mr. McClellan,	„	C.

Radcliffe Observatory, Oxford:
 1901 November 7.

Additional Note on the Green Flash. By W. H. Pickering.

(Communicated by the Secretaries.)

During a recent sea trip I made some further observations upon the green flash at sunset. I was some five or six miles off the Massachusetts coast, and my assistant, Mr. E. R. Cram, and I were watching the Sun set over the land, which was very low at this place. The horizon was fairly clear, and just as the Sun was on the point of disappearing the last ray turned to both of us a bright green. The effect was not so marked, however, as in the sunset I had previously witnessed off the Cuban coast. The next morning Mr. Cram saw the Sun rise out of the ocean, but no unusual colour was observed.

The effect is so marked at sunset that it is singular that it should not have been more generally noticed heretofore. The explanation it seems to me, however, is simple. The effect, I presume, is subjective, and merely a case of complementary colours, like the well-known experiment of gazing fixedly at a red wafer on a white background, then looking away, and seeing a green one in its place. The fact that my assistant could not see it at sunrise, although entirely prepared to do so, confirms this view.

Harvard Observatory:
 1901 October 8.

Ephemeris for Physical Observations of the Moon for 1902.
By A. C. D. Crommelin.

Greenwich Midnight.	1902.	Selenographical Colong. Lat. of the Sun.		Geocentric Libration			Direc- tion.	C.
				Sel. Long. of the Earth.	Lat.	Combined Amount.		
Jan.	1	178°16	+1°33	+5°41	+3°03	6°20	299°3	24°28
	2	190°32	1°34	+4°18	+1°73	4°52	292°5	22°83
	3	202°49	1°35	+2°80	+0°38	2°83	277°7	20°39
	4	214°66	1°36	+1°36	-0°98	1°68	234°2	17°06
	5	226°84	1°37	-0°06	-2°30	2°30	178°5	12°93
	6	239°02	1°38	-1°38	-3°53	3°79	158°6	8°16
	7	251°20	1°39	-2°55	-4°61	5°27	151°1	2°95
	8	263°39	1°40	-3°53	-5°50	6°54	147°3	357°57
	9	275°58	1°41	-4°29	-6°14	7°49	145°1	352°26
	10	287°77	1°42	-4°84	-6°50	8°11	143°3	347°33
	11	299°96	1°43	-5°18	-6°54	8°35	141°6	343°00
	12	312°14	1°44	-5°33	-6°24	8°21	139°5	339°49
	13	324°32	1°45	-5°31	-5°61	7°72	136°6	336°95
	14	336°50	1°46	-5°11	-4°66	6°92	132°4	335°54
	15	348°67	1°47	-4°74	-3°44	5°86	126°0	335°39
	16	0°83	1°48	-4°19	-2°01	4°65	115°6	336°59
	17	12°99	1°48	-3°43	-0°43	3°46	97°1	339°20
	18	25°13	1°49	-2°47	+1°19	2°74	64°3	343°16
	19	37°27	1°50	-1°30	+2°76	3°05	25°2	348°34
	20	49°41	1°51	+0°04	+4°17	4°17	359°4	354°38
	21	61°54	1°52	+1°46	+5°31	5°51	344°6	0°82
	22	73°66	1°52	+2°87	+6°09	6°73	334°8	7°11
	23	85°79	1°53	+4°15	+6°48	7°69	327°4	12°79
	24	97°91	1°53	+5°17	+6°47	8°28	321°4	17°51
	25	110°04	1°53	+5°82	+6°07	8°41	316°2	21°09
	26	122°18	1°53	+6°06	+5°34	8°08	311°4	23°44
	27	134°31	1°53	+5°88	+4°35	7°31	306°5	24°58
	28	146°46	1°53	+5°28	+3°17	6°16	301°0	24°56
	29	158°61	1°53	+4°34	+1°87	4°73	293°3	23°42
	30	170°77	1°52	+3°15	+0°52	3°19	279°4	21°28
	31	182°93	1°52	+1°80	-0°84	1°99	245°0	18°20
Feb.	1	195°10	1°52	+0°38	-2°16	2°19	190°0	14°29
	2	207°27	1°51	-1°00	-3°39	3°53	163°6	9°72
	3	219°45	1°51	-2°26	-4°48	5°02	153°2	4°65
	4	231°63	+1°50	-3°22	-5°39	6°28	149°1	359°30

Greenwich Midnight.	1902.	Selenographical Colong. Lat. of the Sun.		Geocentric Libration				C.
				Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	
Feb.	5	243° 82	+ 1° 50	- 4° 14	- 6° 06	7° 34	145° 7	353° 94
	6	256° 01	1° 49	- 4° 68	- 6° 45	7° 97	144° 0	348° 82
	7	268° 20	1° 49	- 4° 94	- 6° 53	8° 19	142° 9	344° 22
	8	280° 40	1° 48	- 4° 93	- 6° 26	7° 97	141° 8	340° 38
	9	292° 60	1° 48	- 4° 69	- 5° 65	7° 34	140° 3	337° 49
	10	304° 79	1° 47	- 4° 27	- 4° 71	6° 36	137° 8	335° 75
	11	316° 98	1° 47	- 3° 72	- 3° 49	5° 10	133° 2	335° 29
	12	329° 16	1° 47	- 3° 06	- 2° 06	3° 69	124° 0	336° 21
	13	341° 34	1° 46	- 2° 33	- 0° 48	2° 38	101° 6	338° 51
	14	353° 52	1° 46	- 1° 53	+ 1° 13	1° 90	53° 6	342° 16
	15	5° 69	1° 45	- 0° 65	+ 2° 68	2° 76	13° 6	346° 99
	16	17° 85	1° 45	+ 0° 29	+ 4° 07	4° 08	355° 9	352° 70
	17	30° 00	1° 44	+ 1° 28	+ 5° 21	5° 36	346° 2	358° 80
	18	42° 15	1° 43	+ 2° 27	+ 6° 03	6° 44	339° 4	5° 11
	19	54° 29	1° 42	+ 3° 22	+ 6° 48	7° 23	333° 6	10° 89
	20	66° 43	1° 41	+ 4° 05	+ 6° 54	7° 69	328° 2	15° 88
	21	78° 56	1° 40	+ 4° 67	+ 6° 21	7° 78	323° 1	19° 86
	22	90° 70	1° 38	+ 5° 01	+ 5° 55	7° 48	317° 9	22° 68
	23	102° 84	1° 37	+ 5° 03	+ 4° 59	6° 81	312° 1	24° 21
	24	114° 98	1° 35	+ 4° 71	+ 3° 42	5° 82	306° 0	24° 72
	25	127° 13	1° 34	+ 4° 04	+ 2° 11	4° 56	297° 6	23° 97
	26	139° 29	1° 32	+ 3° 09	+ 0° 72	3° 17	283° 1	22° 14
	27	151° 44	1° 30	+ 1° 91	- 0° 67	2° 02	250° 7	19° 34
	28	163° 61	1° 28	+ 0° 59	- 2° 02	2° 10	196° 3	15° 69
Mar.	1	175° 78	1° 27	- 0° 80	- 3° 28	3° 38	166° 3	11° 31
	2	187° 95	1° 25	- 2° 15	- 4° 40	4° 90	154° 0	6° 40
	3	200° 14	1° 23	- 3° 38	- 5° 34	6° 32	147° 7	1° 14
	4	212° 32	1° 21	- 4° 40	- 6° 06	7° 49	144° 0	355° 82
	5	224° 51	1° 20	- 5° 14	- 6° 51	8° 30	141° 7	350° 63
	6	236° 71	1° 18	- 5° 56	- 6° 65	8° 67	140° 1	345° 83
	7	248° 91	1° 16	- 5° 63	- 6° 46	8° 57	138° 9	341° 66
	8	261° 12	1° 14	- 5° 36	- 5° 91	7° 98	137° 8	338° 38
	9	273° 33	1° 13	- 4° 78	- 5° 01	6° 93	136° 3	336° 19
	10	285° 54	1° 11	- 3° 96	- 3° 79	5° 48	133° 7	335° 28
	11	297° 75	1° 10	- 2° 98	- 2° 33	3° 78	128° 0	335° 78
	12	309° 96	1° 08	- 1° 90	- 0° 70	2° 03	110° 2	337° 76
	13	322° 16	1° 07	- 0° 78	+ 0° 97	1° 25	38° 8	341° 16
	14	334° 36	+ 1° 05	+ 0° 31	+ 2° 58	2° 60	353° 1	345° 82

Greenwich Midnight.	Selenographical		Geocentric Libration				C.
	Colong. of the Sun.	Lat.	Sel. Long. of the Earth.	Lat.	Combined Amount.	Direc- tion.	
1902.							
Mar. 15	346°55	+1°03	+1°36	+4°03	4°25	341°4	351°41
16	358°73	1°01	+2°31	+5°21	5°70	336°1	357°52
17	1091	0°99	+3°16	+6°08	6°85	332°5	3°69
18	23°08	0°97	+3°87	+6°57	7°63	329°5	9°51
19	35°24	0°95	+4°44	+6°69	8°03	326°4	14°63
20	47°40	0°93	+4°82	+6°43	8°04	323°1	18°82
21	59°56	0°90	+4°98	+5°82	7°66	319°4	21°95
22	71°71	0°88	+4°90	+4°91	6°93	315°1	23°92
23	83°87	0°85	+4°57	+3°77	5°92	309°5	24°73
24	96°03	0°83	+3°97	+2°46	4°67	301°8	24°37
25	108°19	0°80	+3°13	+1°06	3°30	288°7	22°89
26	120°35	0°77	+2°08	+0°38	2°11	259°6	20°39
27	132°52	0°75	+0°86	-1°77	1°97	205°9	16°97
28	144°69	0°72	-0°48	-3°08	3°12	171°1	12°80
29	156°87	0°69	-1°85	-4°26	4°65	156°5	8°04
30	169°05	0°66	-3°18	-5°25	6°14	148°8	2°91
31	181°24	0°64	-4°40	-6°03	7°46	143°9	357°64
Apr. 1	193°43	0°61	-5°42	-6°55	8°50	140°4	352°43
2	205°63	0°59	-6°17	-6°78	9°17	137°7	347°54
3	217°83	0°56	-6°59	-6°69	9°39	135°4	343°17
4	230°04	0°54	-6°63	-6°25	9°11	133°3	339°57
5	242°26	0°51	-6°27	-5°46	8°31	131°0	336°92
6	254°48	0°49	-5°53	-4°33	7°03	128°1	335°47
7	266°71	0°47	-4°45	-2°90	5°31	123°1	335°39
8	278°93	0°45	-3°12	-1°26	3°36	112°0	336°82
9	291°16	0°42	-1°61	+0°48	1°68	73°4	339°81
10	303°39	0°40	-0°05	+2°20	2°20	1°3	344°21
11	315°61	0°38	+1°47	+3°77	4°05	338°7	349°74
12	327°83	0°35	+2°85	+5°07	5°81	330°7	355°93
13	340°03	0°33	+4°04	+6°04	7°27	326°2	2°26
14	352°23	0°30	+4°96	+6°61	8°27	323°1	8°26
15	4°43	0°28	+5°60	+6°80	8°81	320°5	13°59
16	16°62	0°25	+5°96	+6°60	8°89	317°9	17°98
17	28°80	0°22	+6°03	+6°05	8°54	315°1	21°33
18	40°98	0°19	+5°82	+5°20	7°81	311°8	23°57
19	53°16	0°16	+5°36	+4°10	6°75	307°4	24°65
20	65°34	0°13	+4°67	+2°83	5°47	301°3	24°59
21	77°51	+0°10	+3°77	+1°44	4°04	290°9	23°43

Greenwich Midnight.	Selenographical Longitude. Lat. of the Sun.		Geocentric Libration				C.
			Sel. Long. of the Earth.	Lat.	Combined Amount.	Dirac- tion.	
1902.							
Apr. 22	89°68	+0°07	+2°71	0°00	2°71	270°0	21°24
23	101°86	0°04	+1°51	-1°43	2°08	226°6	18°08
24	114°04	+0°01	+0°22	-2°77	2°78	184°5	14°10
25	126°22	-0°02	-1°14	-4°00	4°16	164°1	9°49
26	138°40	0°05	-2°49	-5°05	5°63	153°8	4°45
27	150°59	0°08	-3°80	-5°88	7°00	147°1	359°21
28	162°78	0°11	-5°00	-6°47	8°18	142°3	354°00
29	174°98	0°14	-6°03	-6°79	9°08	138°4	349°05
30	187°18	0°16	-6°81	-6°79	9°61	134°9	344°58
May 1	199°39	0°19	-7°29	-6°48	9°75	131°6	340°76
2	211°61	0°22	-7°41	-5°82	9°42	128°1	337°80
3	223°83	0°24	-7°12	-4°83	8°60	124°1	335°89
4	236°06	0°26	-6°41	-3°53	7°31	118°8	335°22
5	248°30	0°29	-5°29	-1°98	5°65	110°5	335°99
6	260°54	0°31	-3°82	-0°25	3°83	93°7	338°32
7	272°78	0°33	-2°07	+1°52	2°57	53°7	342°20
8	285°03	0°36	-0°18	+3°19	3°20	3°2	347°43
9	297°27	0°38	+1°72	+4°65	4°96	339°7	353°61
10	309°50	0°40	+3°48	+5°76	6°73	328°9	0°18
11	321°74	0°42	+4°97	+6°48	8°17	322°5	6°54
12	333°96	0°45	+6°11	+6°77	9°12	317°9	12°26
13	346°18	0°47	+6°85	+6°66	9°55	314°2	17°02
14	358°39	0°50	+7°16	+6°17	9°46	310°8	20°67
15	10°60	0°52	+7°09	+5°37	8°89	307°1	23°18
16	22°80	0°55	+6°68	+4°33	7°96	302°9	24°52
17	35°00	0°58	+5°96	+3°09	6°71	297°4	24°73
18	47°19	0°61	+5°02	+1°74	5°31	289°1	23°83
19	59°38	0°64	+3°89	+0°32	3°90	274°7	21°89
20	71°57	0°67	+2°65	-1°09	2°87	247°6	18°99
21	83°76	0°70	+1°33	-2°45	2°79	208°5	15°21
22	95°95	0°72	-0°02	-3°69	3°69	179°7	10°74
23	108°14	0°75	-1°38	-4°77	4°96	163°9	5°78
24	120°33	0°77	-2°69	-5°65	6°26	154°5	0°56
25	132°52	0°80	-3°94	-6°29	7°43	147°9	355°34
26	144°72	0°82	-5°08	-6°66	8°37	142°7	350°31
27	156°92	0°85	-6°05	-6°73	9°05	138°0	345°74
28	169°13	0°87	-6°84	-6°50	9°43	133°5	341°78
29	181°34	-0°89	-7°36	-5°95	9°47	129°0	338°60

Greenwich Midnight.	Selenographical Colong. Lat. of the Sun.		Geocentric Libration				C.
			Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	
1902.							
May 30	193°55	-0°91	-7°57	-5°09	9°12	123°9	336°39
31	205°78	0°93	-7°41	-3°93	8°38	117°9	335°29
June 1	218°01	0°95	-6°86	-2°52	7°31	110°2	335°49
2	230°25	0°96	-5°87	-0°91	5°94	98°8	337°12
3	242°49	0°98	-4°48	+0°81	4°55	79°8	340°29
4	254°74	1°00	-2°73	+2°50	3°70	47°5	344°94
5	266°99	1°01	-0°74	+4°04	4°11	10°4	350°80
6	279°25	1°03	+1°33	+5°30	5°47	345°9	357°37
7	291°50	1°05	+3°32	+6°17	7°01	331°7	4°06
8	303°75	1°07	+5°06	+6°61	8°32	322°6	10°26
9	315°99	1°08	+6°42	+6°61	9°22	315°8	15°56
10	328°23	1°10	+7°30	+6°20	9°57	310°3	19°70
11	340°46	1°12	+7°69	+5°46	9°43	305°4	22°63
12	352°68	1°14	+7°61	+4°45	8°81	300°3	24°31
13	4°90	1°16	+7°13	+3°24	7°83	294°4	24°80
14	17°11	1°18	+6°31	+1°91	6°59	286°8	24°15
15	29°32	1°20	+5°23	+0°53	5°26	275°8	22°46
16	41°52	1°22	+3°99	-0°87	4°08	257°7	19°77
17	53°73	1°24	+2°65	-2°21	3°45	230°2	16°20
18	65°92	1°26	+1°26	-3°45	3°67	200°1	11°90
19	78°12	1°27	-0°10	-4°54	4°54	178°7	7°03
20	90°31	1°29	-1°41	-5°44	5°62	165°5	1°85
21	102°51	1°31	-2°63	-6°10	6°64	156°7	356°57
22	114°70	1°32	-3°76	-6°50	7°51	149°9	351°46
23	126°90	1°34	-4°75	-6°61	8°14	144°3	346°74
24	139°10	1°35	-5°60	-6°42	8°52	138°9	342°63
25	151°31	1°36	-6°28	-5°92	8°63	133°3	339°27
26	163°51	1°37	-6°76	-5°13	8°49	127°2	336°83
27	175°72	1°38	-6°98	-4°06	8°08	120°2	335°46
28	187°94	1°39	-6°91	-2°75	7°51	113°1	335°27
29	200°17	1°40	-6°50	-1°25	6°62	100°9	336°41
30	212°40	1°41	-5°70	+0°36	5°71	86°4	338°95
July 1	224°64	1°41	-4°52	+1°99	4°94	66°2	342°92
2	236°89	1°42	-2°97	+3°52	4°61	40°2	348°21
3	249°14	1°42	-1°13	+4°84	4°97	13°1	354°46
4	261°40	1°43	+0°86	+5°83	5°89	351°6	1°15
5	273°66	1°44	+2°83	+6°40	7°00	336°1	7°68
6	285°91	-1°45	+4°61	+6°53	7°99	324°8	13°51

Greenwich Midnight.	Selenographical Colong. Lat. of the Sun.		Geocentric Libration				C.
			Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	
1902.							
July 7	298°16	-1°45	+6°03	+6°23	8°68	315°9	18°25
8	310°41	1°46	+7°00	+5°55	8°93	308°4	21°74
9	322°65	1°47	+7°47	+4°57	8°76	301°5	23°90
10	334°88	1°48	+7°44	+3°38	8°17	294°4	24°79
11	347°11	1°49	+6°97	+2°05	7°26	286°4	24°47
12	359°33	1°49	+6°14	+0°65	6°17	276°0	23°02
13	11°55	1°50	+5°05	-0°74	5°10	261°7	20°56
14	23°76	1°51	+3°77	-2°08	4°31	241°1	17°20
15	35°97	1°51	+2°41	-3°32	4°10	216°0	13°07
16	48°18	1°52	+1°04	-4°41	4°53	193°3	8°33
17	60°38	1°52	-0°28	-5°31	5°32	177°0	3°20
18	72°58	1°53	-1°51	-5°99	6°18	165°8	357°91
19	84°77	1°53	-2°60	-6°41	6°92	157°9	352°70
20	96°96	1°53	-3°55	-6°54	7°44	151°5	347°83
21	109°16	1°53	-4°33	-6°37	7°70	145°8	343°50
22	121°35	1°53	-4°97	-5°89	7°71	139°8	339°94
23	133°54	1°53	-5°43	-5°12	7°46	133°3	337°27
24	145°74	1°53	-5°72	-4°08	7°03	125°5	335°65
25	157°94	1°53	-5°82	-2°81	6°46	115°8	335°19
26	170°15	1°52	-5°70	-1°36	5°86	103°4	335°99
27	182°36	1°52	-5°31	+0°19	5°31	88°0	338°13
28	194°58	1°51	-4°64	+1°77	4°97	69°1	341°61
29	206°81	1°51	-3°65	+3°26	4°90	48°2	346°35
30	219°05	1°50	-2°36	+4°58	5°15	27°3	352°13
31	231°29	1°49	-0°83	+5°61	5°67	8°4	358°54
Aug. 1	243°53	1°49	+0°86	+6°28	6°34	352°2	5°07
2	255°79	1°48	+2°55	+6°52	7°00	338°6	11°19
3	268°04	1°48	+4°11	+6°33	7°55	327°0	16°42
4	280°29	1°47	+5°39	+5°73	7°86	316°8	20°49
5	292°54	1°47	+6°28	+4°80	7°91	307°4	23°22
6	304°78	1°47	+6°71	+3°61	7°62	298°3	24°62
7	317°02	1°46	+6°69	+2°26	7°06	288°7	24°73
8	329°25	1°46	+6°25	+0°84	6°31	277°7	23°62
9	341°48	1°45	+5°44	-0°59	5°47	263°8	21°43
10	353°70	1°45	+4°38	-1°96	4°80	245°9	18°29
11	5°92	1°44	+3°13	-3°23	4°50	224°1	14°33
12	18°13	1°43	+1°79	-4°35	4°71	202°4	9°75
13	30°33	-1°43	+0°46	-5°28	5°30	185°0	4°71

Greenwich Midnight.	Selenographical Colong. Lat. of the Sun.		Geocentric Libration				C.
			Sel. Long. of the Earth.	Lat.	Combined Amount.	Dirac- tion.	
1902.							
Aug. 14	42°53	-1°42	-0°80	-5°98	6°03	172°4	359°45
15	54°72	1°41	-1°93	-6°43	6°71	163°3	354°18
16	66°92	1°40	-2°88	-6°59	7°19	156°4	349°17
17	79°10	1°39	-3°65	-6°45	7°41	150°5	344°64
18	91°29	1°37	-4°19	-6°00	7°32	145°1	340°80
19	103°47	1°36	-4°55	-5°24	6°94	139°0	337°84
20	115°65	1°34	-4°70	-4°20	6°31	131°8	335°92
21	127°84	1°33	-4°68	-2°92	5°52	122°0	335°17
22	140°03	1°31	-4°49	-1°46	4°72	108°0	335°70
23	152°22	1°29	-4°13	+0°10	4°13	88°6	337°55
24	164°42	1°27	-3°59	+1°68	3°96	64°9	340°72
25	176°62	1°26	-2°87	+3°18	4°29	42°1	345°13
26	188°83	1°24	-1°96	+4°50	4°91	23°5	350°56
27	201°04	1°22	-0°88	+5°56	5°63	9°0	356°68
28	213°27	1°20	+0°35	+6°28	6°29	356°8	3°05
29	225°50	1°19	+1°64	+6°60	6°80	346°0	91°8
30	237°73	1°17	+2°92	+6°50	7°13	335°8	14°63
31	249°97	1°15	+4°08	+6°00	7°26	325°8	19°09
Sept. 1	262°22	1°14	+4°98	+5°14	7°16	315°9	22°32
2	274°46	1°12	+5°59	+3°98	6°87	305°5	24°25
3	286°70	1°11	+5°83	+2°63	6°40	294°3	24°84
4	298°93	1°09	+5°70	+1°18	5°82	281°7	24°17
5	311°16	1°08	+5°21	-0°31	5°22	266°6	22°33
6	323°38	1°06	+4°41	-1°74	4°74	248°5	19°46
7	335°60	1°04	+3°36	-3°07	4°55	227°6	15°72
8	347°81	1°02	+2°15	-4°24	4°75	206°9	11°29
9	0°02	1°01	+0°83	-5°23	5°30	189°0	6°35
10	12°22	0°99	-0°46	-5°99	6°01	175°6	1°14
11	24°41	0°97	-1°68	-6°49	6°70	165°5	355°87
12	36°61	0°95	-2°76	-6°72	7°26	157°7	350°78
13	48°79	0°93	-3°63	-6°64	7°57	151°3	346°07
14	60°97	0°90	-4°25	-6°25	7°56	145°8	341°98
15	73°15	0°88	-4°60	-5°55	7°21	140°3	338°69
16	85°32	0°85	-4°69	-4°53	6°52	134°0	336°39
17	97°49	0°83	-4°51	-3°26	5°57	125°9	335°25
18	109°65	0°80	-4°11	-1°78	4°48	113°4	335°40
19	121°82	0°77	-3°50	-0°17	3°50	92°8	336°93
20	133°99	-0°75	-2°74	+1°47	3°11	61°8	339°84

Greenwich Midn ght.	Selenographical		Geocentric Libration				C.
	Colong. of the Sun.	Lat.	Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	
1902.							
Sept. 21	146° 17	-0° 72	-1° 86	+3° 04	3° 57	31° 5	341° 04
22	158° 35	0° 69	-0° 89	+4° 42	4° 51	11° 4	349° 31
23	170° 53	0° 66	+0° 14	+5° 54	5° 54	358° 6	355° 30
24	182° 73	0° 64	+1° 18	+6° 31	6° 42	349° 4	1° 58
25	194° 93	0° 61	+2° 21	+6° 70	7° 06	341° 7	7° 70
26	207° 14	0° 59	+3° 16	+6° 68	7° 39	334° 7	13° 24
27	219° 35	0° 56	+4° 01	+6° 27	7° 44	327° 4	17° 90
28	231° 57	0° 54	+4° 67	+5° 49	7° 21	319° 6	21° 45
29	243° 79	0° 52	+5° 13	+4° 40	6° 76	310° 6	23° 75
30	256° 01	0° 50	+5° 32	+3° 09	6° 15	300° 2	24° 79
Oct. 1	268° 24	0° 47	+5° 24	+1° 64	5° 49	287° 4	24° 56
2	280° 46	0° 45	+4° 88	+0° 13	4° 88	271° 5	23° 12
3	292° 68	0° 43	+4° 25	-1° 34	4° 46	252° 5	20° 59
4	304° 90	0° 40	+3° 37	-2° 75	4° 35	230° 8	17° 10
5	317° 11	0° 38	+2° 29	-4° 00	4° 61	209° 8	12° 84
6	329° 32	0° 35	+1° 08	-5° 06	5° 18	192° 1	8° 03
7	341° 52	0° 33	-0° 22	-5° 89	5° 89	177° 9	2° 88
8	353° 72	0° 30	-1° 51	-6° 48	6° 65	166° 9	357° 62
9	5° 91	0° 28	-2° 74	-6° 78	7° 31	158° 0	352° 48
10	18° 09	0° 25	-3° 83	-6° 80	7° 81	150° 6	347° 67
11	30° 27	0° 22	-4° 69	-6° 50	8° 01	144° 2	343° 38
12	42° 45	0° 19	-5° 29	-5° 90	7° 93	138° 1	339° 81
13	54° 61	0° 16	-5° 57	-4° 98	7° 47	131° 8	337° 12
14	66° 77	0° 13	-5° 51	-3° 78	6° 68	124° 4	335° 52
15	78° 93	0° 10	-5° 09	-2° 34	5° 60	114° 7	335° 17
16	91° 08	0° 07	-4° 35	-0° 72	4° 41	99° 4	336° 21
17	103° 23	0° 04	-3° 32	+0° 97	3° 46	73° 7	338° 69
18	115° 38	-0° 01	-2° 18	+2° 63	3° 42	39° 7	342° 60
19	127° 53	+0° 03	-0° 72	+4° 12	4° 18	9° 9	347° 73
20	139° 69	0° 06	+0° 68	+5° 35	5° 39	352° 8	353° 73
21	151° 85	0° 09	+2° 02	+6° 23	6° 55	342° 0	0° 11
22	164° 02	0° 12	+3° 23	+6° 70	7° 44	334° 3	6° 38
23	176° 19	0° 15	+4° 24	+6° 76	7° 98	327° 9	12° 11
24	188° 38	0° 17	+5° 03	+6° 42	8° 16	321° 9	16° 95
25	200° 57	0° 20	+5° 55	+5° 72	7° 97	315° 9	20° 72
26	212° 76	0° 22	+5° 82	+4° 70	7° 48	308° 9	23° 31
27	224° 96	0° 25	+5° 84	+3° 46	6° 79	300° 7	24° 66
28	237° 16	+0° 27	+5° 61	+2° 05	5° 97	290° 1	24° 77

Greenwich Midnight.	Selenographical Colong. Lat. of the Sun.		Geocentric Libration				C.
			Sel. Long. of the Earth.	Lat.	Combined Amount.	Direc- tion.	
1902.							
Oct. 29	249°37	+0°30	+5°15	+0°57	5°18	276°3	23°69
30	261°58	0°32	+4°49	-0°92	4°58	258°4	21°51
31	273°78	0°34	+3°64	-2°35	4°33	237°2	18°31
Nov. 1	285°99	0°37	+2°63	-3°65	4°50	215°8	14°28
2	298°19	0°39	+1°48	-4°77	4°99	197°2	9°60
3	310°39	0°42	+0°24	-5°67	5°68	182°4	4°51
4	322°58	0°44	-1°06	-6°32	6°41	170°5	359°37
5	334°77	0°47	-2°37	-6°70	7°11	160°5	354°07
6	346°96	0°49	-3°62	-6°80	7°70	152°0	349°17
7	359°14	0°52	-4°75	-6°60	8°13	144°3	344°76
8	1°131	0°55	-5°69	-6°11	8°35	137°0	340°98
9	2°348	0°58	-6°37	-5°32	8°30	129°9	338°02
10	3°564	0°60	-6°73	-4°25	7°96	122°3	336°02
11	4°780	0°63	-6°69	-2°92	7°30	113°6	335°14
12	5°994	0°66	-6°23	-1°39	6°38	102°6	335°57
13	7°208	0°69	-5°33	+0°27	5°34	87°1	337°43
14	8°422	0°72	-4°04	+1°96	4°49	64°1	340°77
15	9°635	0°75	-2°42	+3°55	4°30	34°3	345°44
16	10°848	0°78	-0°60	+4°91	4°95	7°0	351°41
17	12°062	0°80	+1°27	+5°93	6°07	347°9	357°94
18	13°276	0°83	+3°05	+6°54	7°22	335°0	4°53
19	14°490	0°85	+4°59	+6°70	8°12	325°6	10°65
20	15°705	0°87	+5°79	+6°44	8°67	318°0	15°87
21	16°921	0°90	+6°61	+5°80	8°80	311°3	19°97
22	18°138	0°92	+7°03	+4°84	8°53	304°5	22°85
23	19°356	0°94	+7°08	+3°65	7°97	297°3	24°47
24	20°574	0°96	+6°80	+2°29	7°17	288°6	24°86
25	21°792	0°98	+6°25	+0°85	6°31	277°7	24°08
26	23°010	0°99	+5°48	-0°61	5°51	263°6	22°17
27	24°229	1°01	+4°54	-2°02	4°97	246°0	19°25
28	25°449	1°03	+3°48	-3°32	4°81	226°4	15°46
29	26°668	1°05	+2°32	-4°46	5°03	207°5	10°95
30	27°887	1°07	+1°08	-5°39	5°50	191°3	5°96
Dec. 1	29°106	1°08	-0°20	-6°09	6°09	178°1	0°71
2	30°325	1°10	-1°50	-6°52	6°69	167°0	355°48
3	31°544	1°12	-2°80	-6°67	7°23	157°2	350°48
4	32°762	1°14	-4°04	-6°53	7°68	148°3	345°94
5	33°979	+1°16	-5°20	-6°11	8°02	139°6	342°01

Greenwich Midnight.	Selenographical Colong. Lat. of the Sun.		Geocentric Libration				C.
			Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.	
1902. Dec. 6	351° 97	+ 1° 18	- 6° 21	- 5° 41	8° 24	131° 1	338° 84
7	4° 13	1° 20	- 6° 99	- 4° 45	8° 29	122° 5	336° 57
8	16° 29	1° 22	- 7° 48	- 3° 25	8° 16	113° 5	335° 31
9	28° 44	1° 24	- 7° 60	- 1° 84	7° 82	103° 6	335° 23
10	40° 59	1° 26	- 7° 28	- 0° 29	7° 29	92° 3	336° 44
11	52° 73	1° 28	- 6° 47	+ 1° 33	6° 61	78° 4	339° 07
12	64° 86	1° 29	- 5° 18	+ 2° 91	5° 94	60° 7	343° 15
13	76° 99	1° 31	- 3° 46	+ 4° 34	5° 55	38° 6	348° 56
14	89° 11	1° 33	- 1° 41	+ 5° 49	5° 67	14° 4	354° 93
15	101° 23	1° 35	+ 0° 77	+ 6° 25	6° 30	353° 0	1° 72
16	113° 35	1° 36	+ 2° 91	+ 6° 56	7° 17	336° 1	8° 31
17	125° 48	1° 37	+ 4° 80	+ 6° 41	8° 01	323° 2	14° 13
18	137° 62	1° 39	+ 6° 29	+ 5° 83	8° 58	312° 8	18° 81
19	149° 76	1° 40	+ 7° 31	+ 4° 91	8° 81	303° 9	22° 18
20	161° 91	1° 41	+ 7° 83	+ 3° 73	8° 68	295° 5	24° 18
21	174° 06	1° 41	+ 7° 87	+ 2° 39	8° 23	286° 9	24° 89
22	186° 22	1° 42	+ 7° 50	+ 0° 96	7° 56	277° 3	24° 36
23	198° 39	1° 43	+ 6° 81	- 0° 48	6° 83	266° 0	22° 69
24	210° 56	1° 43	+ 5° 87	- 1° 87	6° 16	252° 3	20° 00
25	222° 74	1° 44	+ 4° 76	- 3° 16	5° 72	236° 4	16° 43
26	234° 92	1° 45	+ 3° 54	- 4° 29	5° 56	219° 5	12° 10
27	247° 10	1° 45	+ 2° 26	- 5° 22	5° 69	203° 4	7° 23
28	259° 29	1° 46	+ 0° 97	- 5° 93	6° 01	189° 3	2° 04
29	271° 47	1° 47	- 0° 33	- 6° 37	6° 38	177° 0	356° 77
30	283° 66	1° 47	- 1° 60	- 6° 55	6° 74	166° 3	351° 68
31	295° 85	1° 48	- 2° 83	- 6° 43	7° 03	156° 2	346° 99
1903. Jan. 1	308° 03	+ 1° 48	- 4° 01	- 6° 04	7° 25	146° 4	342° 88

The longitudes are reckoned in the plane of the Moon's equator, the axis of reference being the radius which passes through the mean centre of the visible disc. This axis therefore rotates with the Moon, and is not fixed in space.

The inclination of the Moon's equator to the ecliptic is taken as 1° 523, the value used in the *Connaissance des Temps*, that given by the *Nautical Almanac* being 1° 536.

The physical librations in longitude and latitude, as given by Professor Franz's formulæ, have been applied; their values are taken from the *Berliner Jahrbuch* for the days given there, and interpolated by a graphical method for the other days. But the signs in the *Jahrbuch* require to be reversed in order to reduce to the system used here.

The colongitude of the Sun is 90° (or 450°) *minus* his selenographical longitude. It also is the selenographical longitude of the morning terminator reckoned eastward from the mean centre of the disc. Hence its value is approximately $270^\circ, 0^\circ, 90^\circ, 180^\circ$ at new Moon, first quarter, full Moon, last quarter respectively. The longitude of the evening terminator is of course 180° greater or less than that of the morning one.

When the geocentric libration in longitude is positive, the region brought into view is on the west limb ; when negative, on the east.

When the geocentric libration in latitude is positive, the region brought into view is at the Moon's north pole ; when negative, at the south.

The column "Combined Amount" gives the distance between the apparent and mean centres of the disc, and the column "Direction" gives the position-angle of the apparent centre from the mean centre, or, which is the same thing, the position-angle of the region which is most carried into view by libration. The angles are reckoned eastward from the northern extremity of the Moon's axis.

C denotes the geocentric position-angle of the northern extremity of the Moon's axis measured eastward from the northernmost point of the disc. It has been computed by the second formula given in the Preface to the *Nautical Almanac*, but the co-ordinates of the Moon's equator have been taken from the *Connaissance des Temps*, so as to make this column consistent with the rest of the ephemeris.

The terms "East" and "West" are used throughout with reference to our sky, and not as they would appear to an observer on the Moon.

I give the method for finding the altitude of the Sun at a given point on the Moon whose position is defined : (1) by selenographical longitude and latitude ; (2) by direction cosines.

In either case the Sun's selenographical colongitude and latitude (K, L supposed) must be found by interpolation from the ephemeris for the given time.

In the first case let the given point be in the position longitude M, latitude N. Longitudes are reckoned from the meridian passing through the mean centre of the disc, and the positive direction is that towards Mare Crisium. North latitudes are considered positive.

Then

$$\text{sine Sun's altitude} = \sin L \sin N + \cos L \cos N \sin (K + M).$$

In the second case let ξ, η, ζ be the direction cosines of the given point. The axes are (1) that diameter of the Moon's equator which is 90° from the mean centre of the disc ; (2) the Moon's polar axis ; (3) the diameter through the mean centre of the disc. The positive directions are as above. Mr. Saunder

has issued some maps of portions of the Moon's surface from which the co-ordinates ξ , η , ζ can be taken at sight.

Then the Sun's direction cosines are :

$$\cos K \cos L, \sin L, \sin K \cos L,$$

and sine Sun's altitude

$$= \xi \cos K \cos L + \eta \sin L + \zeta \sin K \cos L.$$

Neither formula is convenient when the Sun's altitude is very great, for an angle near 90° cannot be accurately determined from its sine. However, when the Sun is high the shadows are so inconspicuous that it is not necessary to compute his altitude with great accuracy.

Benvenue, 55 Ulundi Road, Blackheath, S.E.
1901 September 21.

Ephemeris for Physical Observations of

Greenwich Noon.	P.	L-O.	B.	Apparent Diameter.			d.	Q.	B'.
				Equat. 2a.	Defect. 2b.	Polar 2b.			
1902.									
Feb. 21	344°765	167°836	-0°822	33'46	0'07	31'31	5°31	255°90	-0°88
23	344°604	168°275	-0°803	33'55	'08	31'39	5°57	255°78	-0°86
25	344°446	168°712	-0°785	33'65	'09	31'48	5°84	255°66	-0°84
27	344°290	169°145	-0°767	33'74	'10	31'57	6°10	255°54	-0°82
Mar. 1	344°137	169°573	-0°749	33'85	'10	31'67	6°36	255°42	-0°80
3	343°987	169°999	-0°731	33'96	'11	31'77	6°61	255°31	-0°78
5	343°839	170°420	-0°713	34'07	'12	31'87	6°86	255°19	-0°76
7	343°694	170°837	-0°695	34'19	'13	31'98	7°11	255°08	-0°74
9	343°551	171°249	-0°677	34'31	'14	32'10	7°35	254°97	-0°72
11	343°410	171°656	-0°659	34'44	'15	32'22	7°59	254°86	-0°70
13	343°272	172°059	-0°641	34'58	'16	32'35	7°82	254°75	-0°68
15	343°137	172°456	-0°623	34'72	'17	32'48	8°04	254°64	-0°67
17	343°004	172°849	-0°605	34'86	'18	32'61	8°27	254°54	-0°65
19	342°874	173°234	-0°587	34'99	'19	32°74	8°48	254°43	-0°63
21	342°748	173°615	-0°570	35'14	'20	32°87	8°69	254°33	-0°61
23	342°624	173°990	-0°552	35°29	'21	33°02	8°89	254°23	-0°59
25	342°502	174°358	-0°535	35°45	'22	33°17	9°09	254°13	-0°57
27	342°383	174°720	-0°517	35°62	'23	33°33	9°28	254°04	-0°55
29	342°268	175°076	-0°500	35°79	'24	33°49	9°46	253°95	-0°53
31	342°156	175°425	-0°483	35°96	'25	33°65	9°64	253°86	-0°52
Apr. 2	342°046	175°766	-0°466	36°14	'26	33°82	9°81	253°77	-0°50
4	341°939	176°102	-0°449	36°33	'27	33°99	9°97	253°68	-0°48
6	341°835	176°428	-0°432	36°52	'28	34°17	10°13	253°59	-0°46
8	341°734	176°748	-0°415	36°71	'29	34°35	10°28	253°50	-0°44
10	341°637	177°060	-0°398	36°91	'30	34°53	10°42	253°42	-0°43
12	341°543	177°363	-0°381	37°12	'31	34°72	10°55	253°34	-0°41
14	341°452	177°659	-0°365	37°32	'32	34°92	10°67	253°27	-0°39
16	341°364	177°946	-0°349	37°53	'33	35°12	10°79	253°20	-0°37
18	341°279	178°223	-0°333	37°75	'34	35°32	10°89	253°13	-0°36
20	341°197	178°492	-0°317	37°97	'34	35°52	10°99	253°06	-0°34
22	341°119	178°753	-0°301	38°19	'35	35°73	11°08	253°00	-0°32
24	341°044	179°002	-0°286	38°41	'36	35°94	11°15	252°94	-0°31
26	340°972	179°244	-0°271	38°64	'37	36°16	11°22	252°89	-0°29
28	340°903	179°474	-0°256	38°88	'37	36°38	11°28	252°84	-0°27
30	340°837	179°696	-0°242	39°12	0°38	36°60	11°33	252°80	-0°26

Jupiter, 1902-3. By A. C. D. Crommelin.

Greenwich Noon.	Longitude of Υ 's Central Meridian.		Corr. for Phase.	Light- time. m	$\Delta-O.$	$B.$
	877° 90 I.	870° 27 II.				
1902.						
Feb. 21	170° 69	346° 30	+ 0° 12	49° 645	162° 528	- 0° 920
23	126° 13	286° 48	'14	49° 513		
25	81° 58	226° 67	'15	49° 375		
27	37° 04	166° 86	'16	49° 232		
Mar. 1	352° 50	107° 06	'18	49° 079		
3	307° 97	47° 27	'19	48° 921	163° 382	- 0° 877
5	263° 45	347° 49	'20	48° 759		
7	218° 94	287° 72	'22	48° 592		
9	174° 43	227° 95	'24	48° 419		
11	129° 93	168° 19	'25	48° 240		
13	85° 44	108° 44	'27	48° 055	164° 239	- 0° 833
15	40° 96	48° 70	'28	47° 866		
17	356° 49	348° 97	'30	47° 672		
19	312° 02	289° 24	'31	47° 474		
21	267° 56	229° 52	'33	47° 271		
23	223° 11	169° 81	'34	47° 062	165° 097	- 0° 788
25	178° 67	110° 11	'36	46° 849		
27	134° 24	50° 42	'38	46° 632		
29	89° 82	350° 74	'39	46° 413		
31	45° 41	291° 07	'40	46° 189		
Apr. 2	1° 01	231° 40	'42	45° 958	165° 956	- 0° 744
4	316° 62	171° 75	'43	45° 725		
6	272° 24	112° 11	'45	45° 489		
8	227° 86	52° 47	'46	45° 250		
10	183° 49	352° 84	'47	45° 008		
12	139° 14	293° 22	'48	44° 763	166° 816	- 0° 699
14	94° 80	233° 62	'49	44° 515		
16	50° 47	174° 03	'51	44° 265		
18	6° 14	114° 44	'52	44° 013		
20	321° 82	54° 86	'53	43° 759		
22	277° 52	355° 30	'54	43° 502	167° 676	- 0° 654
24	233° 23	295° 75	'54	43° 242		
26	188° 95	236° 21	'55	42° 982		
28	144° 68	176° 68	'55	42° 722		
30	100° 41	117° 15	+ 0° 56	42° 460		

Greenwich Noon.	P.	L-O.	B.	Apparent Diameter.		d.	Q.	B'.
				Equat. 2a.	Defect. Polar 2b.			
1902.								
May 2	340°775	179°908	-0°228	39'37	0'38 36''83	11°37	252°75	-0°24
4	340°716	180°109	-0°214	39'62	'39 37'06	11°40	252°71	-0°23
6	340°660	180°298	-0°200	39'87	'39 37'29	11°42	252°67	-0°21
8	340°608	180°477	-0°187	40'12	'40 37'53	11°42	252°64	-0°20
10	340°559	180°645	-0°174	40'38	'40 37'77	11°42	252°61	-0°19
12	340°513	180°801	-0°161	40'64	'40 38'01	11°40	252°59	-0°17
14	340°471	180°947	-0°148	40'90	'40 38'26	11°37	252°57	-0°16
16	340°432	181°081	-0°136	41'16	'40 38'51	11°33	252°56	-0°14
18	340°397	181°203	-0°124	41'43	'40 38'76	11°28	252°54	-0°13
20	340°365	181°312	-0°113	41'70	'40 39'01	11°22	252°53	-0°12
22	340°337	181°410	-0°102	41'97	'40 39'26	11°14	252°52	-0°11
24	340°312	181°496	-0°091	42'25	'39 39'51	11°06	252°51	-0°10
26	340°291	181°569	-0°080	42'52	'39 39'77	10°96	252°51	-0°09
28	340°273	181°631	-0°070	42'79	'38 40'03	10°85	252°52	-0°08
30	340°258	181°679	-0°060	43'06	'38 40'29	10°72	252°52	-0°06
June 1	340°247	181°715	-0°051	43'34	'37 40'55	10°59	252°53	-0°05
3	340°240	181°739	-0°042	43'62	'36 40°81	10°44	252°55	-0°04
5	340°236	181°749	-0°034	43'89	'35 41°07	10°27	252°58	-0°04
7	340°236	181°746	-0°026	44'16	'34 41°32	10°10	252°62	-0°03
9	340°239	181°731	-0°018	44'43	'33 41°57	9°91	252°66	-0°02
11	340°246	181°703	-0°011	44'70	'32 41°82	9°71	252°71	-0°01
13	340°257	181°663	-0°004	44'97	'31 42°07	9°49	252°76	0°00
15	340°271	181°608	+0°002	45'23	'30 42°32	9°26	252°82	0°00
17	340°289	181°543	+0°007	45'49	'29 42°56	9°02	252°89	+0°01
19	340°310	181°465	+0°012	45'74	'27 42°80	8°77	252°97	+0°01
21	340°335	181°374	+0°017	45'99	'26 43°03	8°51	253°05	+0°02
23	340°364	181°272	+0°021	46'24	'25 43°26	8°23	253°13	+0°02
25	340°395	181°158	+0°025	46'49	'23 43°49	7°94	253°22	+0°03
27	340°430	181°031	+0°029	46'72	'21 43°71	7°64	253°31	+0°03
29	340°468	180°893	+0°032	46'95	'19 43°92	7°33	253°41	+0°03
July 1	340°509	180°744	+0°034	47'17	'18 44°12	7°01	253°53	+0°04
3	340°554	180°585	+0°036	47'38	'16 44°32	6°68	253°66	+0°04
5	340°602	180°415	+0°038	47'58	'14 44°51	6°33	253°81	+0°04
7	340°652	180°235	+0°039	47'77	'13 44°70	5°98	253°97	+0°04
9	340°706	180°045	+0°040	47'95	'12 44°87	5°61	254°15	+0°04
11	340°763	179°846	+0°040	48'12	'10 45°03	5°24	254°36	+0°04
13	340°823	179°638	+0°039	48'29	'09 45°18	4°86	254°59	+0°04
15	340°886	179°423	+0°038	48'45	0°07 45°33	4°47	254°84	+0°04

Greenwich Noon.	Longitude of Λ 's Central Meridian.		Corr. for Phase.	Light- time.	$\Lambda-O.$	$B.$
	877° 90' L.	870° 27' IL.				
1902.				^m		
May 2	56° 16'	57° 63'	+ 0° 56'	42° 197	168° 539	- 0° 609
4	11° 92'	358° 13'	° 56'	41° 933		
6	327° 69'	298° 64'	° 57'	41° 670		
8	283° 47'	239° 16'	° 57'	41° 407		
10	239° 26'	179° 69'	° 57'	41° 144		
12	195° 07'	120° 23'	° 56'	40° 881	169° 401	- 0° 563
14	150° 88'	60° 78'	° 56'	40° 618		
16	106° 71'	1° 35'	° 56'	40° 356		
18	62° 55'	301° 93'	° 55'	40° 097		
20	18° 40'	242° 52'	° 55'	39° 839		
22	334° 26'	183° 11'	° 54'	39° 581	170° 266	- 0° 518
24	290° 13'	123° 72'	° 53'	39° 325		
26	246° 01'	64° 34'	° 52'	39° 072		
28	201° 90'	4° 97'	° 51'	38° 823		
30	157° 80'	305° 61'	° 50'	38° 576		
June 1	113° 71'	246° 26'	° 49'	38° 331	171° 131	- 0° 472
3	69° 63'	186° 92'	° 47'	38° 090		
5	25° 56'	127° 59'	° 46'	37° 853		
7	341° 51'	68° 27'	° 44'	37° 619		
9	297° 47'	8° 96'	° 43'	37° 390		
11	253° 44'	309° 67'	° 41'	37° 165	172° 000	- 0° 426
13	209° 41'	250° 38'	° 39'	36° 945		
15	165° 39'	191° 10'	° 37'	36° 730		
17	121° 38'	131° 83'	° 35'	36° 520		
19	77° 39'	72° 58'	° 33'	39° 316		
21	33° 40'	13° 34'	° 31'	36° 117	172° 867	- 0° 380
23	349° 42'	314° 10'	° 29'	35° 925		
25	305° 45'	254° 87'	° 28'	35° 740		
27	261° 49'	195° 65'	° 26'	35° 561		
29	217° 53'	136° 43'	° 24'	35° 389		
July 1	173° 58'	77° 21'	° 21'	35° 223	173° 736	- 0° 333
3	129° 63'	18° 00'	° 19'	35° 065		
5	85° 69'	318° 80'	° 17'	34° 915		
7	41° 76'	259° 61'	° 16'	34° 773		
9	357° 84'	200° 42'	° 14'	34° 639		
11	313° 91'	141° 23'	° 12'	34° 513	174° 606	- 0° 287
13	269° 99'	82° 05'	° 10'	34° 395		
15	226° 07'	22° 87'	+ 0° 09'	34° 288		

Greenwich No. n.	P.	L—O.	B.	Apparent Diameter.			d.	Q.	B'.
				Equat. 2a.	Defect. 2b.	Polar 2b.			
1902.									
July 17	340° 952	179° 200	+0° 037	48° 59	0° 06	45° 46	4° 07	255° 14	+0° 04
19	341° 020	178° 972	+0° 035	48° 72	° 05	45° 58	3° 67	255° 50	+0° 04
21	341° 091	178° 735	+0° 033	48° 84	° 04	45° 69	3° 26	255° 93	+0° 04
23	341° 163	178° 494	+0° 030	48° 94	° 03	45° 79	2° 84	256° 45	+0° 03
25	341° 237	178° 248	+0° 027	49° 03	° 02	45° 87	2° 42	257° 12	+0° 03
27	341° 313	177° 998	+0° 024	49° 10	° 01	45° 94	2° 00	258° 16	+0° 03
29	341° 390	177° 743	+0° 020	49° 16	° 01	46° 00	1° 58	259° 57	+0° 02
31	341° 467	177° 486	+0° 016	49° 21	° 01	46° 05	1° 15	263° 72	+0° 02
Aug. 2	341° 546	177° 227	+0° 012	49° 25	° 00	46° 08	0° 72	268° 54	+0° 01
4	341° 625	176° 967	+0° 008	49° 27	° 00	46° 10	0° 32	287° 32	+0° 00
6	341° 706	176° 706	+0° 003	49° 28	° 00	46° 10	0° 25	27° 59	0° 00
8	341° 787	176° 445	—0° 002	49° 27	° 00	46° 10	0° 63	57° 32	0° 00
10	341° 867	176° 186	—0° 007	49° 24	° 00	46° 07	1° 06	64° 08	—0° 01
12	341° 948	175° 928	—0° 012	49° 20	° 01	46° 04	1° 49	66° 98	—0° 01
14	342° 028	175° 674	—0° 017	49° 15	° 01	45° 99	1° 91	68° 62	—0° 02
16	342° 109	175° 422	—0° 023	49° 08	° 02	45° 93	2° 33	69° 72	—0° 02
18	342° 188	175° 174	—0° 029	49° 00	° 03	45° 86	2° 76	70° 47	—0° 03
20	342° 266	174° 932	—0° 035	48° 90	° 04	45° 77	3° 17	71° 04	—0° 04
22	342° 342	174° 694	—0° 041	48° 80	° 05	45° 67	3° 59	71° 51	—0° 04
24	342° 417	174° 463	—0° 048	48° 69	° 06	45° 56	3° 99	71° 88	—0° 05
26	342° 490	174° 238	—0° 054	48° 56	° 07	45° 44	4° 39	72° 19	—0° 06
28	342° 561	174° 022	—0° 060	48° 41	° 09	45° 30	4° 78	72° 47	—0° 06
30	342° 630	173° 812	—0° 066	48° 25	° 10	45° 15	5° 17	72° 72	—0° 07
Sept. 1	342° 696	173° 611	—0° 072	48° 09	° 11	44° 99	5° 54	72° 93	—0° 08
3	342° 760	173° 420	—0° 078	47° 92	° 13	44° 83	5° 91	73° 13	—0° 08
5	342° 821	173° 237	—0° 083	47° 74	° 15	44° 65	6° 27	73° 31	—0° 09
7	342° 879	173° 066	—0° 089	47° 54	° 16	44° 46	6° 62	73° 48	—0° 10
9	342° 933	172° 904	—0° 094	47° 33	° 18	44° 27	6° 96	73° 62	—0° 10
11	342° 983	172° 754	—0° 099	47° 11	° 19	44° 07	7° 28	73° 75	—0° 11
13	343° 030	172° 615	—0° 104	46° 89	° 21	43° 87	7° 60	73° 88	—0° 11
15	343° 073	172° 488	—0° 109	46° 66	° 22	43° 65	7° 90	74° 01	—0° 12
17	343° 112	172° 372	—0° 114	46° 42	° 23	43° 43	8° 19	74° 13	—0° 12
19	343° 147	172° 269	—0° 119	46° 18	° 24	43° 21	8° 47	74° 24	—0° 13
21	343° 178	172° 178	—0° 123	45° 94	° 26	42° 98	8° 73	74° 34	—0° 13
23	343° 205	172° 100	—0° 127	45° 69	° 28	42° 75	8° 99	74° 43	—0° 14
25	343° 227	172° 034	—0° 131	45° 44	° 29	42° 51	9° 23	74° 50	—0° 14
27	343° 245	171° 981	—0° 135	45° 18	° 30	42° 27	9° 46	74° 56	—0° 14
29	343° 258	171° 941	—0° 138	44° 92	0° 32	42° 02	9° 67	74° 61	—0° 15

Greenwich Noon. 1902.	Longitude of 2l's Central Meridian.		Corr. for Phase.	Light- time. m	$\Delta-O.$	B.
	877° 0' 90 I.	870° 27' II.				
July 17	182° 15'	323° 69'	+ 0° 07'	34° 190		
19	138° 24'	264° 52'	'06	34° 100		
21	94° 32'	205° 34'	'05	34° 019	175° 478	+ 0° 241
23	50° 40'	146° 16'	'04	33° 947		
25	6° 49'	86° 99'	'03	33° 884		
27	322° 57'	27° 81'	'02	33° 831		
29	278° 65'	328° 63'	'01	33° 788		
31	234° 73'	269° 45'	+ '01	33° 754	176° 350	- 0° 194
Aug. 2	190° 80'	210° 26'	'00	33° 730		
4	146° 87'	151° 07'	'00	33° 716		
6	102° 93'	91° 88'	'00	33° 712		
8	58° 99'	32° 67'	'00	33° 718		
10	15° 04'	333° 46'	'00	33° 734	177° 223	- 0° 146
12	331° 08'	274° 24'	'01	33° 760		
14	287° 11'	215° 01'	'02	33° 796		
16	243° 14'	155° 78'	'02	33° 842		
18	199° 16'	96° 54'	'03	33° 897		
20	155° 16'	37° 28'	'04	33° 961	178° 097	- 0° 099
22	111° 15'	338° 01'	'06	34° 035		
24	67° 13'	278° 73'	'07	34° 118		
26	23° 10'	219° 44'	'08	34° 212		
28	339° 06'	160° 14'	'10	34° 312		
30	295° 00'	100° 82'	'12	34° 422	178° 972	- 0° 052
Sept. 1	250° 93'	41° 49'	'13	34° 541		
3	206° 84'	342° 15'	'15	34° 669		
5	162° 74'	282° 79'	'17	34° 804		
7	118° 62'	223° 41'	'19	34° 948		
9	74° 49'	164° 02'	'21	35° 100	179° 849	- 0° 005
11	30° 34'	104° 61'	'23	35° 259		
13	346° 18'	45° 19'	'25	35° 426		
15	302° 00'	345° 75'	'27	35° 601		
17	257° 81'	286° 30'	'29	35° 780		
19	213° 60'	226° 83'	'31	35° 967	180° 727	+ 0° 042
21	169° 37'	167° 34'	'33	36° 159		
23	125° 12'	107° 84'	'35	36° 360		
25	80° 86'	48° 32'	'37	36° 565		
27	36° 59'	348° 79'	'39	36° 775		
29	352° 30'	289° 24'	- 0° 41	36° 991	181° 607	+ 0° 089

Greenwich Noon.	P	L-O.	B.	Apparent Diameter.			d.	Q.	B'.
				Equat. 2 ^h .	Defect. 2 ^h .	Polar 2 ^h .			
1902.									
Oct. 1	343°267	171°915	-0°141	44°65	0°33	41°77	9°88	74°65	-0°15
3	343°272	171°902	-0°143	44°38	°34	41°52	10°06	74°69	-0°15
5	343°272	171°902	-0°145	44°11	°35	41°27	10°24	74°72	-0°16
7	343°268	171°915	-0°147	43°84	°36	41°02	10°40	74°75	-0°16
9	343°259	171°942	-0°148	43°56	°37	40°76	10°55	74°77	-0°16
11	343°246	171°982	-0°149	43°29	°37	40°51	10°69	74°79	-0°16
13	343°228	172°035	-0°150	43°02	°38	40°25	10°81	74°81	-0°16
15	343°206	172°101	-0°150	42°75	°38	39°99	10°92	74°83	-0°16
17	343°179	172°181	-0°150	42°48	°39	39°74	11°02	74°84	-0°16
19	343°148	172°273	-0°149	42°21	°39	39°48	11°10	74°84	-0°16
21	343°113	172°376	-0°148	41°94	°39	39°23	11°17	74°83	-0°16
23	343°074	172°494	-0°147	41°67	°40	38°98	11°23	74°82	-0°16
25	343°031	172°623	-0°145	41°40	°40	38°73	11°28	74°81	-0°16
27	342°983	172°765	-0°143	41°14	°40	38°49	11°32	74°80	-0°15
29	342°931	172°919	-0°140	40°88	°40	38°25	11°34	74°78	-0°15
31	342°876	173°085	-0°137	40°62	°40	38°01	11°35	74°75	-0°15
Nov. 2	342°817	173°263	-0°133	40°36	°40	37°77	11°35	74°72	-0°14
4	342°754	173°451	-0°129	40°11	°39	37°54	11°34	74°69	-0°14
6	342°688	173°652	-0°125	39°86	°39	37°30	11°31	74°66	-0°13
8	342°618	173°864	-0°121	39°62	°38	37°07	11°28	74°62	-0°13
10	342°545	174°087	-0°116	39°38	°38	36°84	11°23	74°58	-0°12
12	342°469	174°320	-0°110	39°14	°37	36°62	11°17	74°54	-0°12
14	342°390	174°563	-0°104	38°91	°37	36°40	11°11	74°49	-0°11
16	342°308	174°817	-0°098	38°68	°36	36°19	11°03	74°44	-0°10
18	342°223	175°081	-0°091	38°46	°35	35°98	10°94	74°38	-0°10
20	342°136	175°354	-0°084	38°24	°34	35°77	10°85	74°32	-0°09
22	342°046	175°636	-0°076	38°02	°33	35°57	10°74	74°26	-0°08
24	341°953	175°929	-0°068	37°81	°33	35°37	10°63	74°20	-0°07
26	341°858	176°230	-0°059	37°60	°32	35°18	10°50	74°14	-0°06
28	341°761	176°539	-0°050	37°40	°31	34°99	10°37	74°08	-0°05
30	341°662	176°858	-0°041	37°20	°30	34°81	10°23	74°01	-0°04
Dec. 2	341°561	177°185	-0°032	37°01	°29	34°63	10°08	73°95	-0°03
4	341°458	177°519	-0°022	36°82	°28	34°45	9°92	73°88	-0°02
6	341°353	177°862	-0°012	36°64	°26	34°28	9°76	73°82	-0°01
8	341°247	178°212	-0°002	36°46	°25	34°11	9°58	73°75	0°00
10	341°140	178°570	+0°009	36°29	°24	33°95	9°40	73°68	+0°01
12	341°031	178°934	+0°020	36°12	°23	33°79	9°21	73°62	+0°02
14	340°921	179°306	+0°032	35°96	0°22	33°64	9°02	73°55	+0°03

Greenwich Noon.	Longitude of Υ 's Central Meridian.		Corr. for Phase.	Light- time.	$\Delta-O.$	$B.$
	877° 90' I.	870° 27' II.				
1902.				m		
Oct. 1	307° 99	229° 67	-0° 42	37° 213	°	°
3	263° 67	170° 09	44	37° 436		
5	219° 33	110° 49	46	37° 665		
7	174° 98	50° 88	47	37° 898		
9	130° 61	351° 25	48	38° 135	182° 486	+0° 136
11	86° 22	291° 60	50	38° 375		
13	41° 81	231° 93	51	38° 616		
15	357° 39	172° 25	52	38° 864		
17	312° 97	112° 58	53	39° 114		
19	268° 53	52° 88	54	39° 364	183° 367	+0° 183
21	224° 08	353° 17	55	39° 617		
23	179° 61	293° 45	55	39° 870		
25	135° 12	233° 70	55	40° 125		
27	90° 61	173° 94	56	40° 382		
29	46° 10	114° 16	56	40° 639	184° 249	+0° 230
31	1° 58	54° 38	56	40° 897		
Nov. 2	317° 05	354° 59	56	41° 155		
4	272° 50	294° 78	56	41° 413		
6	227° 94	234° 96	56	41° 671		
8	183° 37	175° 13	55	41° 928	185° 131	+0° 275
10	138° 79	115° 29	55	42° 185		
12	94° 20	55° 44	54	42° 440		
14	49° 60	355° 58	54	42° 694		
16	4° 99	295° 72	53	42° 947		
18	320° 38	235° 85	52	43° 198	186° 014	+0° 322
20	275° 76	175° 97	51	43° 447		
22	231° 12	116° 07	50	43° 693		
24	186° 48	56° 17	49	43° 939		
26	141° 83	356° 26	48	44° 180		
28	97° 18	296° 35	47	44° 419	186° 898	+0° 370
30	52° 52	236° 43	46	44° 654		
Dec. 2	7° 85	176° 50	44	44° 887		
4	323° 17	116° 57	43	45° 116		
6	278° 49	56° 63	41	45° 342		
8	233° 81	356° 69	40	45° 564	187° 784	+0° 417
10	189° 12	296° 74	38	45° 781		
12	144° 42	236° 79	37	45° 995		
14	99° 72	176° 83	-0° 35	46° 205		

Greenwich Noon.	P.	L-O.	B.	Apparent Diameter.			δ .	Q.	B'.
				Equat. 2a.	Defect. 2b.	Polar 2c.			
1902. Dec. 16	340°8'10	179°6'83	+0°0'44	35°8'0	0°21	33°49	8°8'2	73°48'	+0°0'5
18	340°6'97	180°0'68	+0°0'56	35°6'4	'20	33°35	8°6'1	73°41'	+0°0'6
20	340°5'84	180°4'58	+0°0'69	35°4'9	'19	33°21	8°4'0	73°35'	+0°0'7
22	340°4'70	180°8'55	+0°0'82	35°3'5	'18	33°07	8°1'8	73°28'	+0°0'9
24	340°3'56	181°2'57	+0°0'96	35°2'1	'17	32°94	7°9'6	73°22'	+0°1'0
26	340°2'42	181°6'65	+0°1'10	35°0'8	'16	32°82	7°7'3	73°16'	+0°1'2
28	340°1'27	182°0'79	+0°1'25	34°9'5	'15	32°70	7°4'9	73°10'	+0°1'3
30	340°0'12	182°4'97	+0°1'40	34°8'2	'14	32°58	7°2'5	73°05'	+0°1'5
1903. Jan. 1	339°8'97	182°9'21	+0°1'55	34°7'0	'13	32°47	7°0'0	73°01'	+0°1'7
3	339°7'81	183°3'50	+0°1'70	34°5'9	'12	32°36	6°7'5	72°96'	+0°1'8
5	339°6'65	183°7'81	+0°1'86	34°4'8	'11	32°26	6°5'0	72°92'	+0°2'0
7	339°5'50	184°2'19	+0°2'02	34°3'8	'10	32°16	6°2'4	72°87'	+0°2'2
9	339°4'35	184°6'59	+0°2'19	34°2'8	'09	32°07	5°9'8	72°83'	+0°2'3
11	339°3'19	185°1'04	+0°2'36	34°1'9	'08	31°98	5°7'1	72°78'	+0°2'5
13	339°2'04	185°5'52	+0°2'53	34°1'0	'08	31°90	5°4'4	72°74'	+0°2'7
15	339°0'88	186°0'03	+0°2'71	34°0'1	'07	31°82	5°1'7	72°69'	+0°2'9
17	338°9'72	186°4'59	+0°2'89	33°9'3	0°06	31°74	4°8'9	72°65'	+0°3'1

The following is a list of the Greenwich Mean Times when the Zero meridians of the two adopted systems cross the centre of the illuminated disc :—

System I.

1902. d	h	m	1902. d	h	m	1902. d	h	m	1902. d	h	m		
Feb. 21	5	10'38	Feb. 27	18	40'19	Mar. 6	8	9'86	Mar. 12	21	39'39		
	15	1'00		28	4	30'80		18	0'46		13	7	29'98
22	0	51'62		14	21'41	7	3	51'06		17	20'57		
	10	42'24	Mar. 1	0	12'02		13	41'66		14	3	11'17	
	20	32'85		10	2'63		23	32'26		13	1'76		
23	6	23'47		19	53'24	8	9	22'86		22	52'35		
	16	14'08		2	5	43'85		19	13'45		15	8	42'94
24	2	4'69		15	34'45	9	5	4'05		18	33'53		
	11	55'30		3	1	25'05		14	54'64		16	4	24'12
	21	45'91		11	15'66	10	0	45'23		14	14'71		
25	7	36'52		21	6'26		10	35'83		17	0	5'30	
	17	27'13		4	6	56'86		20	26'43		9	55'89	
26	3	17'74		16	47'46	11	6	17'02		19	46'47		
	13	8'35		5	2	38'06		16	7'61		18	5	37'05
	22	58'97		12	28'66	12	1	58'21		15	27'63		
27	8	49'58		22	19'26		11	48'80		19	1	18'21	

Greenwich Noon.	Longitude of Λ 's Central Meridian.		Corr. for Phase.	Light- time.	$\Lambda-O.$	$B.$
1902.	877° 90 I.	870° 27 II.		m.		
Dec. 16	55° 02	116° 87	-0° 34	46.410		
18	10° 31	56° 90	.32	46.610	188.671	+0.465
20	325° 60	356° 93	.31	46.804		
22	280° 89	296° 96	.29	46.995		
24	236° 18	236° 99	.28	47.180		
26	191° 46	177° 01	.26	47.360		
28	146° 74	117° 03	.25	47.534	189.559	+0.512
30	102° 02	57° 05	.23	47.705		
1903.						
Jan. 1	57° 30	357° 06	.21	47.868		
3	12° 57	297° 08	.20	48.026		
5	327° 84	237° 09	.18	48.178		
7	283° 12	177° 11	.17	48.324	190.448	+0.559
9	238° 40	117° 12	.16	48.464		
11	193° 67	57° 14	.14	48.598		
13	148° 95	357° 15	.13	48.726		
15	104° 22	297° 17	.12	48.848		
17	59° 49	237° 19	-0° 10	48.963	191.338	+0.607

System I.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
Mar. 19 11 8.79	Mar. 26 20 19.17	Apr. 3 5 29.35	Apr. 10 14 39.33
20 59.37	27 6 9.74	15 19.91	11 0 29.88
20 6 49.96	16 0.31	4 1 10.47	10 20.42
16 40.54	28 1 50.88	11 1.03	20 10.96
21 2 31.12	11 41.45	20 51.60	12 6 1.51
12 21.70	21 32.03	5 6 42.16	15 52.06
22 12.28	29 7 22.59	16 32.72	13 1 42.60
22 8 2.86	17 13.15	6 2 23.29	11 33.14
17 53.44	30 3 3.72	12 13.85	21 23.68
23 3 44.01	12 54.28	22 4.42	14 7 14.23
13 34.59	22 44.84	7 7 54.98	17 4.77
23 25.16	31 8 35.40	17 45.53	15 2 55.31
24 9 15.73	18 25.96	8 3 36.07	12 45.85
19 6.31	Apr. 1 4 16.53	13 26.62	22 36.40
25 4 56.88	14 7.09	23 17.16	16 8 26.94
14 47.45	23 57.65	9 9 7.70	18 17.48
26 0 38.02	2 9 48.22	18 58.25	17 4 8.02
10 28.63	19 38.78	10 4 48.79	13 58.56

System I.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
Apr. 17 23 49.09	May 3 13 49.01	May 19 3 47.96	June 3 17 45.95
18 9 39.62	23 39.52	13 38.44	4 3 36.41
19 30.16	4 9 30.03	23 28.93	13 26.87
19 5 20.69	19 20.54	20 9 19.41	23 17.33
15 11.22	5 5 11.05	19 9.89	5 9 7.79
20 1 1.75	15 1.56	21 5 0.37	18 58.25
10 52.29	6 0 52.07	14 50.85	6 4 48.70
20 42.82	10 42.58	22 0 41.33	14 39.15
21 6 33.35	20 33.09	10 31.81	7 0 29.60
16 23.88	7 6 23.60	20 22.29	10 20.05
22 2 14.41	16 14.11	23 6 12.77	20 10.50
12 4.94	8 2 4.61	16 3.24	8 6 0.95
21 55.47	11 55.11	24 1 53.72	15 51.40
23 7 46.01	21 45.61	11 44.20	9 1 41.85
17 36.54	9 7 36.11	21 34.68	11 32.30
24 3 27.07	17 26.61	25 7 25.15	21 22.75
13 17.60	10 3 17.11	17 15.63	10 7 13.20
23 8.14	13 7.61	26 3 6.11	17 3.65
25 8 58.67	22 58.11	12 56.59	11 2 54.10
18 49.20	11 8 48.61	22 47.07	12 44.55
26 4 39.73	18 39.11	27 8 37.54	22 35.00
14 30.26	12 4 29.61	18 28.01	12 8 25.45
27 0 20.79	14 20.11	28 4 18.48	18 15.90
10 11.32	13 0 10.61	14 8.95	13 4 6.35
20 1.84	10 1.10	23 59.42	13 56.80
28 5 52.35	19 51.60	29 9 49.89	23 47.25
15 42.86	14 5 42.09	19 40.35	14 9 37.70
29 1 33.37	15 32.59	30 5 30.82	19 28.15
11 23.89	15 1 23.07	15 21.29	15 5 18.59
21 14.40	11 13.56	31 1 11.76	15 9.03
30 7 4.91	21 4.05	11 2.22	16 0 59.47
16 55.42	16 6 54.54	20 52.69	10 49.91
May 1 2 45.94	16 45.04	June 1 6 43.16	20 40.35
12 36.45	17 2 35.54	16 33.63	17 6 30.79
22 26.96	12 26.04	2 2 24.10	16 21.23
2 8 17.47	22 16.52	12 14.57	18 2 11.67
18 7.98	18 8 7.01	22 5.03	12 2.11
3 3 58.50	17 57.49	3 7 55.49	21 52.55

System I.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
June 19 7 42.99	July 4 21 39.17	July 20 11 34.82	Aug. 5 1 30.35
17 33.42	5 7 29.59	21 25.23	11 20.76
20 3 23.85	17 20.01	21 7 15.64	21 11.17
13 14.28	6 3 10.42	17 6.05	6 7 1.59
23 4.72	13 0.84	22 2 56.46	16 52.00
21 8 55.15	22 51.25	12 46.87	7 2 42.41
18 45.58	7 8 41.67	22 37.28	12 32.83
22 4 36.01	18 32.08	23 8 27.69	22 23.24
14 26.44	8 4 22.50	18 18.10	8 8 13.66
23 0 16.87	14 12.91	24 4 8.51	18 4.07
10 7.30	9 0 3.33	13 58.92	9 3 54.49
19 57.73	9 53.74	23 49.33	13 44.91
24 5 48.16	19 44.16	25 9 39.74	23 35.33
15 38.59	10 5 34.57	19 30.14	10 9 25.75
25 1 29.02	15 24.98	26 5 20.54	19 16.17
11 19.45	11 1 15.39	15 10.94	11 5 6.59
21 9.88	11 5 8.0	27 1 1.34	14 57.01
26 7 0.31	20 56.21	10 51.74	12 0 47.43
16 50.74	12 6 46.62	20 42.15	10 37.86
27 2 41.17	16 37.03	28 6 32.56	20 28.28
12 31.60	13 2 27.44	16 22.97	13 6 18.70
22 22.03	12 17.85	29 2 13.38	16 9.13
28 8 12.45	22 8.26	12 3.79	14 1 59.55
18 2.87	14 7 58.67	21 54.20	11 49.97
29 3 53.29	17 49.08	30 7 44.61	21 40.40
13 43.71	15 3 39.49	17 35.02	15 7 30.82
23 34.13	13 29.90	31 3 25.43	17 21.25
30 9 24.55	23 20.31	13 15.84	16 13 11.68
19 14.97	16 9 10.72	23 6.25	13 2.11
July 1 5 5.39	19 1.13	Aug. 1 8 56.66	22 52.54
14 55.81	17 4 51.54	18 47.07	17 8 42.97
2 0 46.23	14 41.95	2 4 37.48	18 33.40
10 36.65	18 0 32.36	14 27.89	18 4 23.84
20 27.07	10 22.77	3 0 18.30	14 14.27
3 6 17.49	20 13.18	10 8.71	19 0 4.71
16 7.91	19 6 3.59	19 59.12	9 55.14
4 1 58.33	15 54.00	4 5 49.53	19 45.58
11 48.75	20 1 44.41	15 39.94	20 5 36.02

System I.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
Aug. 20 15 26.46	Sept. 5 5 23.84	Sept. 20 19 22.71	Oct. 6 9 23.16
21 1 16.90	15 14.32	21 5 13.23	19 13.72
11 7.34	6 1 4.80	15 3.75	7 5 4.28
20 57.78	10 55.28	22 0 54.28	14 54.84
22 6 48.23	20 45.76	10 44.80	8 0 45.40
16 38.67	7 6 36.24	20 35.32	10 35.96
23 2 29.11	16 26.72	23 6 25.85	20 26.52
12 19.56	8 2 17.20	16 16.37	9 6 17.09
22 10.00	12 7.68	24 2 6.89	16 7.66
24 8 0.45	21 58.16	11 57.42	10 1 58.23
17 50.89	9 7 48.65	21 47.95	11 48.80
25 3 41.34	17 39.14	25 7 38.48	21 39.37
13 31.79	10 3 29.63	17 29.01	11 7 29.94
23 22.24	13 20.13	26 3 19.55	17 20.52
26 9 12.69	23 10.62	13 10.08	12 3 11.09
19 3.15	11 9 1.11	23 0.61	13 1.66
27 4 53.60	18 51.61	27 8 51.15	22 52.23
14 44.06	12 4 42.10	18 41.69	13 8 42.81
28 0 34.51	14 32.60	28 4 32.22	18 33.38
10 24.97	13 0 23.09	14 22.76	14 4 23.96
20 15.43	10 13.59	29 0 13.30	14 14.53
29 6 5.89	20 4.09	10 3.84	15 0 5.11
15 56.35	14 5 54.59	19 54.38	9 55.69
30 1 46.81	15 45.09	30 5 44.93	19 46.27
11 37.27	15 1 35.59	15 35.47	16 5 36.85
21 27.74	11 26.09	Oct. 1 1 26.02	15 27.43
31 7 18.20	21 16.60	11 16.56	17 1 18.02
17 8.67	16 7 7.10	21 7.11	11 8.60
Sept. 1 2 59.13	16 57.60	2 6 57.65	20 59.18
12 49.60	17 2 48.10	16 48.19	18 6 49.77
22 40.06	12 38.61	3 2 38.74	16 40.35
2 8 30.53	22 29.12	12 29.29	19 2 30.94
18 21.00	18 8 19.63	22 19.84	12 21.53
3 4 11.47	18 10.14	4 8 10.39	22 12.12
14 1.94	19 4 0.65	18 0.94	20 8 2.71
23 52.42	13 51.16	5 3 51.50	17 53.31
4 9 42.89	23 41.67	13 42.05	21 3 43.90
19 33.37	20 9 32.19	23 32.60	13 34.49

System I.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
Oct. 21 23 25.08	Nov. 6 13 28.19	Nov. 22 3 32.28	Dec. 7 17 37.04
22 9 15.68	23 18.81	13 22.92	8 3 27.70
19 6.27	7 9 9.43	23 13.57	13 18.36
23 4 56.86	19 0.05	23 9 4.21	23 9.02
14 47.46	8 4 50.68	18 54.85	9 8 59.68
24 0 38.06	14 41.31	24 4 45.50	18 50.34
10 28.66	9 0 31.94	14 36.14	10 4 41.00
20 19.26	10 22.57	25 0 26.79	14 31.66
25 6 9.86	20 13.20	10 17.44	11 0 22.32
16 0.47	10 6 3.83	20 8.09	10 12.99
26 1 51.07	15 54.46	26 5 58.73	20 3.65
11 41.68	11 1 45.09	15 49.38	12 5 54.31
21 32.28	11 35.72	27 1 40.03	15 44.98
27 7 22.88	21 26.35	11 30.68	13 1 35.64
17 13.48	12 7 16.99	21 21.33	11 26.30
28 3 4.09	17 7.62	28 7 11.98	21 16.97
12 54.69	13 2 58.25	17 2.63	14 7 7.63
22 45.29	12 48.88	29 2 53.28	16 58.29
29 8 35.90	22 39.51	12 43.94	15 2 48.96
18 26.51	14 8 30.14	22 34.59	12 39.62
30 4 17.12	18 20.78	30 8 25.24	22 30.29
14 7.73	15 4 11.41	18 15.90	16 8 20.95
23 58.34	14 2.05	Dec. 1 4 6.55	18 11.62
31 9 48.96	23 52.68	13 57.20	17 4 2.28
19 39.57	16 9 43.32	23 47.86	13 52.95
Nov. 1 5 30.18	19 33.95	2 9 38.51	23 43.61
15 20.79	17 5 24.58	19 29.17	18 9 34.28
2 1 11.41	15 15.22	3 5 19.82	19 24.94
11 2.02	18 1 5.86	15 10.48	19 5 15.61
20 52.64	10 56.50	4 1 1.13	15 6.27
3 6 43.25	20 47.14	10 51.79	20 0 56.94
16 33.86	19 6 37.78	20 42.45	10 47.60
4 2 24.48	16 28.42	5 6 33.10	20 38.27
12 15.09	20 2 19.07	16 23.76	21 6 28.93
22 5.71	12 9.71	6 2 14.41	16 19.60
5 7 56.33	22 0.35	12 5.07	22 2 10.27
17 46.95	21 7 50.99	21 55.73	12 0.94
3 37.57	17 41.64	7 7 46.38	21 51.60

System I.

1902. d h m	1902. d h m	1903. d h m	1903. d h m
Dec. 23 7 42.27	Dec. 29 21 12.99	Jan. 5 0 53.06	Jan. 11 14 23.81
17 32.94	30 7 3.66	10 43.73	12 0 14.48
24 3 23.61	16 54.33	20 34.40	10 5.16
13 14.28	31 2 45.00	6 6 25.07	19 55.83
23 4.95	12 35.67	16 15.74	13 5 46.50
25 8 55.62	22 26.34	7 2 6.42	15 37.18
18 46.29	1903. Jan. 1 8 17.01	11 57.09	14 1 27.85
26 4 36.96	18 7.68	21 47.76	11 18.52
14 27.63	2 3 58.35	8 7 38.43	21 9.19
27 0 18.30	13 49.03	17 29.10	15 6 59.87
10 8.97	23 39.70	9 3 19.78	16 50.54
19 59.64	3 9 30.37	13 10.45	16 2 41.21
28 5 50.31	19 21.04	23 1.12	12 31.89
15 40.98	4 5 11.71	10 8 51.79	22 22.56
29 1 31.65	15 2.39	18 42.46	17 8 13.23
11 22.32		11 4 33.14	

System II.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
Feb. 21 0 22.48	Mar. 1 6 58.30	Mar. 9 13 33.91	Mar. 17 20 9.29
10 18.27	16 54.09	23 29.69	18 6 5.06
20 14.07	2 2 49.88	10 9 25.46	16 0.82
22 6 9.86	12 45.67	19 21.24	19 1 56.58
16 5.65	22 41.46	11 5 17.01	11 52.34
23 2 1.44	3 8 37.24	15 12.79	21 48.10
11 57.24	18 33.02	12 1 8.56	20 7 43.86
21 53.03	4 4 28.80	11 4.33	17 39.62
24 7 48.82	14 24.58	21 0.10	21 3 35.38
17 44.61	5 0 20.36	13 6 55.87	13 31.14
25 3 40.40	10 16.14	16 51.64	23 26.90
13 36.19	20 11.91	14 2 47.41	22 9 22.66
23 31.98	6 6 7.69	12 43.17	19 18.42
26 9 27.77	16 3.47	22 38.94	23 5 14.18
19 23.56	7 1 59.25	15 8 34.70	15 9.93
27 5 19.35	11 55.02	18 30.47	24 1 5.68
15 15.14	21 50.80	16 4 26.23	11 1.43
28 1 10.93	8 7 46.58	14 22.00	20 57.18
11 6.72	17 42.36	17 0 17.76	25 6 52.93
21 2.51	9 3 38.13	10 13.53	16 48.68

System II.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
Mar. 26 2 44.43	Apr. 10 20 2.53	Apr. 26 13 19.63	May 12 6 35.81
12 40.17	11 5 58.26	23 15.33	16 31.48
22 35.92	15 53.99	27 9 11.03	13 2 27.15
27 8 31.67	12 1 49.71	19 6.73	12 22.82
18 27.42	11 45.43	28 5 2.43	22 18.48
28 4 23.17	21 41.15	14 58.13	14 8 14.15
14 18.91	13 7 36.87	29 0 53.83	18 9.82
29 0 14.66	17 32.58	10 49.53	15 4 5.48
10 10.41	14 3 28.30	20 45.23	14 1.15
20 6.16	13 24.02	30 6 40.93	23 56.82
30 6 1.90	23 19.74	16 36.62	16 9 52.48
15 57.65	15 9 15.45	May 1 2 32.32	19 48.15
31 1 53.39	19 11.17	12 28.01	17 5 43.82
11 49.14	16 5 6.88	22 23.71	15 39.48
21 44.88	15 2.60	2 8 19.40	18 1 35.15
Apr. 1 7 40.63	17 0 58.32	18 15.09	11 30.82
17 36.38	10 54.03	3 4 10.78	21 26.48
2 3 32.12	20 49.75	14 6.47	19 7 22.15
13 27.86	18 6 45.46	4 0 2.15	17 17.81
23 23.60	16 41.18	9 57.84	20 3 13.48
3 9 19.33	19 2 36.89	19 53.53	13 9.14
19 15.07	12 32.61	5 5 49.21	23 4.80
4 5 10.80	22 28.32	15 44.89	21 9 0.47
15 6.54	20 8 24.03	6 1 40.58	18 56.13
5 1 2.27	18 19.75	11 36.26	22 4 51.79
10 58.01	21 4 15.46	21 31.95	14 47.45
20 53.74	14 11.17	7 7 27.63	23 0 43.10
6 6 49.48	22 0 6.88	17 23.32	10 38.76
16 45.21	10 2.59	8 3 19.00	20 34.41
7 2 40.95	19 58.29	13 14.69	24 6 30.07
12 36.58	23 5 54.00	23 10.38	16 25.72
22 32.42	15 49.70	9 9 6.07	25 2 21.38
8 8 28.15	24 1 45.41	19 1.75	12 17.03
18 23.89	11 41.11	10 4 57.43	22 12.68
9 4 19.62	21 36.82	14 53.11	26 8 8.33
14 15.35	25 7 32.52	11 0 48.79	18 3.98
10 0 11.08	17 28.23	10 44.46	27 3 59.63
10 6.80	26 3 23.93	20 40.14	13 55.28

System II.

1902. d	h	m	1902. d	h	m	1902. d	h	m	1902. d	h	m
May 27	23	50.93	June 12	17	5.08	June 28	10	18.31	July 14	3	30.86
28	9	46.58	13	3	0.70	20	13.91		13	26.44	
19	42.22		12	56.32	29	6	9.51		23	22.03	
29	5	37.87	22	51.94	16	5.11		15	9	17.61	
15	33.51		14	8	47.56	30	2	0.71		19	13.20
30	1	29.16	18	43.18	11	56.31		16	5	8.78	
11	24.80		15	4	38.80	21	51.91		15	4.37	
21	20.45		14	34.42	July 1	7	47.51		17	0	59.95
31	7	16.09	16	0	30.04	17	43.11		10	55.54	
17	11.74		10	25.66	2	3	38.70		20	51.12	
June 1	3	7.38	20	21.27	13	34.30		18	6	46.71	
13	3.02		17	6	16.89	23	29.89		16	42.29	
22	58.66		16	12.50	3	9	25.49		19	2	37.88
2	8	54.30	18	2	8.12	19	21.08		12	33.46	
18	49.93		12	3.73	4	5	16.68		22	29.05	
3	4	45.57	21	59.35	15	12.27		20	8	24.63	
14	41.21		19	7	54.96	5	1	7.87		18	20.21
4	0	36.84	17	50.58	11	3.46		21	4	15.79	
10	32.48		20	3	46.19	20	59.06		14	11.37	
20	28.11		13	41.81	6	6	54.65		22	0	6.95
5	6	23.75	23	37.42	16	50.24		10	2	5.4	
16	19.38		21	9	33.03	7	2	45.84		19	58.12
6	2	15.02	19	28.64	12	41.43		23	5	53.70	
12	10.65		22	5	24.25	22	37.02		15	49.29	
22	6.29		15	19.86	8	8	32.61		24	1	44.87
7	8	1.92	23	1	15.46	18	28.20		11	40.45	
17	57.55		11	11.07	9	4	23.79		21	36.04	
8	3	53.18	21	6.68	14	19.38		25	7	31.62	
13	48.81		24	7	2.28	10	0	14.97		17	27.20
23	44.44		16	57.88	10	10.56		26	3	22.79	
9	9	40.07	25	2	53.49	20	6.15		13	18.37	
19	35.70		12	49.10	11	6	1.74		23	13.95	
10	5	31.33	22	44.71	15	57.33		27	9	9.54	
15	26.96		26	8	40.31	12	1	52.92		19	5.12
11	1	22.59	18	35.91	11	48.51		28	5	0.70	
11	18.21		27	4	31.51	21	44.10		14	56.29	
21	13.84		14	27.11	13	7	39.68		29	0	51.87
12	7	9.46	28	0	22.71	17	35.27		10	47.45	

System II.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
July 29 20 43.04	Aug. 14 13 55.53	Aug. 30 7 9.02	Sept. 15 0 24.02
30 6 38.62	23 51.13	17 4.66	10 19.70
16 34.20	15 9 46.74	31 3 0.30	20 15.38
31 2 29.79	19 42.34	12 55.95	16 6 11.07
12 25.37	16 5 37.94	22 51.59	16 6.75
22 20.95	15 33.54	Sept. 1 8 47.24	17 2 2.44
Aug. 1 8 16.54	17 1 29.14	18 42.88	11 58.12
18 12.12	11 24.75	2 4 38.53	21 53.81
2 4 7.70	21 20.35	14 34.17	18 7 49.49
14 3.29	18 7 15.95	3 0 29.82	17 45.18
23 58.87	17 11.55	10 25.46	19 3 40.87
3 9 54.46	19 3 7.16	20 21.11	13 36.56
19 50.05	13 2.78	4 6 16.75	23 32.25
4 5 45.63	22 58.39	16 12.41	20 9 27.95
15 41.22	20 8 54.01	5 2 8.06	19 23.64
5 1 36.81	18 49.63	12 3.72	21 5 19.34
11 32.39	21 4 45.25	21 59.37	15 15.04
21 27.98	14 40.87	6 7 55.03	22 1 10.73
6 7 23.57	22 0 36.49	17 50.68	11 6.43
17 19.16	10 32.12	7 3 46.34	21 2.13
7 3 14.75	20 27.74	13 41.99	23 6 57.83
7 13 10.34	23 6 23.37	23 37.65	16 53.52
23 5.93	16 18.99	8 9 33.30	24 2 49.22
8 9 1.53	24 2 14.61	19 28.96	12 44.93
18 57.12	12 10.24	9 5 24.62	22 40.64
9 4 52.72	22 5.86	15 20.28	25 8 36.35
14 48.31	25 8 1.48	10 1 15.95	18 32.06
10 0 43.91	17 57.11	11 11.61	26 4 27.77
10 39.51	26 3 52.73	21 7.28	14 23.48
20 35.11	13 48.35	11 7 2.95	27 0 19.19
11 6 30.71	23 43.98	16 58.62	10 14.91
16 26.32	27 9 39.60	12 2 54.30	20 10.62
12 2 21.92	19 35.23	12 49.97	28 6 6.33
12 17.52	28 5 30.85	22 45.65	16 2.05
22 13.12	15 26.48	13 8 41.32	29 1 57.77
13 8 8.73	29 1 22.11	18 37.00	11 53.49
18 4.33	11 17.75	14 4 32.67	21 49.21
14 3 59.93	21 13.38	14 28.35	30 7.44.93

System II.

1902. d h m	1902. d h m	1902. d h m	1902. d h m
Sept. 30 17 40'66	Oct. 16 10 58'83	Nov. 1 4 18'32	Nov. 16 21 38'89
Oct. 1 3 36'38	20 54'59	14 14'11	17 7 34'70
13 32'10	17 6 50'34	2 0 9'90	17 30'51
23 27'83	16 46'10	10 5'70	18 3 26'33
2 9 23'54	18 2 41'86	20 1'49	13 22'15
19 19'26	12 37'62	3 5 57'28	23 17'97
3 5 14'99	22 33'38	15 53'07	19 9 13'79
15 10'71	19 8 29'15	4 1 48'87	19 9'62
4 1 6'43	18 24'92	11 44'66	20 5 5'44
11 2'16	20 4 20'69	21 40'46	15 1'26
20 57'89	14 16'46	5 7 36'25	21 0 57'08
5 6 53'63	21 0 12'24	17 32'05	10 52'91
16 49'36	10 8'01	6 3 27'85	20 48'73
6 2 45'09	20 3'78	13 23'65	22 6 44'55
12 40'83	22 5 59'56	23 19'45	16 40'38
22 36'56	15 55'33	7 9 15'26	23 2 36'20
7 8 32'30	23 1 51'11	19 11'06	12 32'03
18 28'04	11 46'88	8 5 6'87	22 27'85
8 4 23'78	21 42'65	15 2'67	24 8 23'6
14 19'52	24 7 38'43	9 0 58'48	18 19'50
9 0 15'27	17 34'20	10 54'29	25 4 15'32
10 11'02	25 3 29'98	20 50'10	14 11'15
20 6'77	13 25'75	10 6 45'91	26 0 6'98
10 6 2'52	23 21'53	16 41'72	10 2'81
15 58'27	26 9 17'31	11 2 37'53	19 58'63
11 1 54'03	19 13'08	12 33'34	27 5 54'46
11 49'78	27 5 8'86	22 29'15	15 50'29
21 45'53	15 4'64	12 8 24'96	28 1 46'12
12 7 41'29	28 1 0'42	18 20'77	11 41'95
17 37'04	10 56'21	13 4 16'58	21 37'78
13 3 32'79	20 51'99	14 12'39	29 7 33'61
13 28'55	29 6 47'78	14 0 8'20	17 29'44
23 24'30	16 43'57	10 4'02	30 3 25'27
14 9 20'06	30 2 39'36	19 59'83	13 21'1
19 15'81	12 35'15	15 5 55'64	23 16'94
15 5 11'57	22 30'95	15 51'45	Dec. 1 9 12'77
15 7'32	31 8 26'74	16 1 47'27	19 8'61
16 1 3'08	18 22'53	11 43'08	2 5 4'44

System II.

1902. d h m	1902. d h m	1902. d h m	1903. d h m
Dec. 2 15 0.28	Dec. 14 5 3.77	Dec. 25 19 7.46	Jan. 5 23 15.43
3 0 56.11	14 59.61	26 5 3.31	6 9 11.28
10 51.95	15 0 55.45	14 59.16	19 7.13
20 47.78	10 51.29	27 0 55.01	7 5 2.99
4 6 43.61	20 47.13	10 50.86	14 58.84
16 39.45	16 6 42.98	20 46.71	8 0 54.69
5 2 35.28	16 38.82	28 6 42.56	10 50.54
12 31.12	17 2 34.67	16 38.41	20 46.39
22 26.95	12 30.51	29 2 34.26	9 6 42.24
6 8 22.79	22 26.35	12 30.11	16 38.10
18 18.63	18 8 22.20	22 25.96	10 2 33.95
7 4 14.46	18 18.04	30 8 21.81	12 29.80
14 10.30	19 4 13.89	18 17.66	22 25.65
8 0 6.14	14 9.74	31 4 13.51	11 8 21.50
10 1.98	20 0 5.58	14 9.37	18 17.36
19 57.82	10 1.43	1903. Jan. 1 0 5.22	12 4 13.21
9 5 53.66	19 57.28	10 1.07	14 9.06
15 49.50	21 5 53.12	19 56.92	13 0 4.91
10 1 45.35	15 48.97	2 5 52.77	10 0.76
11 41.19	22 1 44.82	15 48.62	19 56.61
21 37.03	11 40.67	3 1 44.47	14 5 52.46
11 7 32.87	21 36.52	11 40.32	15 48.32
17 28.71	23 7 32.36	21 36.17	15 1 44.17
12 3 24.56	17 28.21	4 7 32.02	11 40.02
13 20.40	24 3 24.06	17 27.88	21 35.87
23 16.24	13 19.91	5 3 23.73	16 7 31.72
13 9 12.08	23 15.76	13 19.58	17 27.68
19 7.92	25 9 11.61		17 3 23.43

The quantities in the ephemeris are to be interpolated directly for the times for which they are required, the equation of light having been already applied.

The position of *Jupiter's* North Pole is assumed to be R.A. $17^h 51^m 59^s.49$, N.P.D. $25^\circ 26' 25''.3$ at the beginning of 1902, and R.A. $17^h 51^m 59^s.75$, N.P.D. $25^\circ 26' 26''.0$ at the beginning of 1903.

P denotes the position-angle of the northern extremity of *Jupiter's* axis, reckoned eastward from the northernmost point of the disc.

L-O + 180° , Δ -O + 180° are the jovicentric right ascensions of the Earth and Sun respectively, reckoned in the plane of the

planet's equator from O, the point of the vernal equinox of *Jupiter's* northern hemisphere; B, *B* are the jovicentric declinations of the Earth and Sun above the planet's equator.

The adopted values of the diameters at distance 5.20 are: Equatorial 38''.419; Polar 35''.945. The sources from which these values are derived were given last year.

The assumed time for light to traverse the unit distance is 498^s.92, this being the same value as that used by Mr. Marth.

d denotes the jovicentric angle between the Earth and Sun.

Q denotes the position-angle of the point of greatest phase, and is reckoned eastward from the northernmost point of the disc. It also gives the position-angle of the shadows of the satellites measured from the satellites themselves.

B' is obtained from *B* by the formula $\tan B' = \sec \epsilon_0 \tan B$, where $\sec \epsilon_0 = \frac{a}{b} = \frac{15.53}{14.53}$. Since *B*, *B'* can never exceed some

3°, we may take the tangents as proportional to the angles. Hence it suffices in practice to find *B'* by the formula

$B' = \frac{15.53}{14.53} B$. *B'* is the eccentric angle of the centre of the

disc. If we call *B''* the jovigraphical latitude of the centre of the disc, then we can find *B''* by the formulæ:

$$\tan B' = \sec^2 \epsilon_0 \tan B \quad \text{or} \quad \tan B'' = \sec \epsilon_0 \tan B'.$$

The longitudes of *Jupiter's* central meridian are computed with unaltered values of the rates of rotation and of the zero-meridians in the two adopted systems. The addition of the "Corr. for Phase" gives the longitudes of the meridians which bisect the illuminated disc.

The sidereal periods of rotation corresponding to the two adopted systems are 9^h 50^m 30^s.004, 9^h 55^m 40^s.632.

Contrary to expectation, the longitude of the Red Spot seems to have remained nearly stationary during the last apparition. I have received the following determinations of its longitude relative to the zero-meridian of System II.

(D.=W. F. Denning, A.=E. M. Antoniadi, P.=Rev. T. E. R. Phillips.)

Date. 1901	Obs.	Long.	Date. 1901	Obs.	Long.	Date. 1901	Obs.	Long.
Feb. 13	D.	43°5	June 22	A.	43° (uncertain)	July 8	D.	45°2
May 1	"	46°4	24	D.	44°7	28	"	46°54
23	"	43°6	26	A.	47°2	Aug. 21	P.	46°4
28	"	45°9	26	D.	47°1	Sept. 24	"	46°3
June 14	"	45°9	July 4	"	44°6	...	...	...

These figures show that its longitude has scarcely varied since 1900 October, when it was estimated by A. to be 45°.

A. reports a remarkable swelling of the south Tropical Belt

on September 30, making the Belt nearly touch the south Temperate Band. The longitude of this swelling (System II.) was about 194° .

Observers are reminded that the Jovian system will be presented edgewise in 1902, and a watch should be kept for mutual occultations and eclipses of the satellites.

It will be remembered that the late Mr. Marth attached particular value to measurements of the latitudes of their shadows while crossing the disc of *Jupiter*, for the purpose of improving our knowledge of the positions of their orbit-planes.

A list of times of elongation of the fifth satellite is given in the *Connaissance des Temps* for 1902.

It may be mentioned here that the *Connaissance des Temps* for 1899 and following years gives ephemerides for the satellites of *Mars*, *Saturn*, *Uranus* and *Neptune* in the same form as those formerly contributed to the *Monthly Notices* by Mr. Marth.

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